UPPSALA UNIVERSITET
Matematiska institutionen
Erik Palmgren

PROBLEM 1
Tillämpad Logik DV1, ht-01
2001-09-18

Problem set 1

The following problems are to be solved and presented by you at the black board on September 25. At the beginning of the lesson sign up for the problems you have solved and be prepared! Satisfactory solution of at least 3 of the problems gives 1 bonus point to be counted in the final exam.

1. Prove the principle of excluded middle (PEM)
$$A \vee \neg A$$
for an arbitrary formula A in natural deduction for classical logic (i.e. intutiontistic rules + RAA). Conversely, prove RAA in intutionistic logic assuming PEM as an axiom scheme.

2. Prove by constructing suitable Kripke models that none of the following formulae are provable in intuitionistic logic:
   (a) $\sim\sim P \supset P$,
   (b) $\sim(P \wedge Q) \supset (\sim P \vee \sim Q)$.

3. Define the binary natural numbers as a recursive data type. E.g. in postfix notation: $(((\varepsilon)1)0)0$, $((((\varepsilon)0)1)0)0$ both represent 4, ε represent 0. Provide introduction, elimination and computation rules in the style of Martin-Löf type theory.

4. It is known that the Ackermann function $a : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, defined by
$$\begin{aligned} a(0,n) &= S(n) \\ a(S(m),0) &= a(m,S(0)) \\ a(S(m),S(n)) &= a(m,a(S(m),n)), \end{aligned}$$
grows too quickly to be a primitive recursive function. (Compute it for some small arguments.) Show that it nevertheless can be defined in the typed lambda calculus presented in "Konstruktiv logik" using the recursion operator. (Hint 1: first expand $a(S(m),n)$ using the definition. Hint 2: define a function $b : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$ such that $a(m,n) = b(m)(n)$.)

5. Let $\mathcal{M}$ be a modal logic model where the accessibility relation is a linear order. Let $u \in \{\Box, \Diamond\}^*$ denote an arbitrary string of modal operators. If A is a modal formula, then $u\,A$ is a modal formula. Show using Exercise 4.5 that $u\,A$ is is equivalent to one of the formulae
$$A \qquad \Box A \qquad \Diamond A \qquad \Box\Diamond A \qquad \Diamond\Box A.$$

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