UPPSALA UNIVERSITET
MATEMATISKA INSTITUTIONEN
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Complex Analysis 10hp
Kand Ma2
Spring Term 2014

Home assignment 2
(for training only - do not hand in)

1. (a) Compute $\int_C \frac{1}{z^3-1}\,dz$, where C is the positively (counter-clock) oriented circle $|z+\frac{1}{2}|=1$.

(b) Compute $\int_\Gamma \frac{z}{(z^2-1)^2}\,dz$, where Γ is the positively oriented circle $|z|=2$.

2. Suppose that $f(z)$ is analytic in a region Ω and γ is a closed curve in Ω. Show that

$$\int_\gamma \overline{f(z)}f'(z)\,dz$$

is purely imaginary.

3. Assume that $f(z)$ is analytic and satisfies the inequality $|f(z)-1|<1$ in a region Ω. Show that

$$\int_\gamma \frac{f'(z)}{f(z)}\,dz=0$$

for every closed curve γ in Ω.

4. If $P(z)$ is a polynomial and C denotes the positively oriented circle $|z-a|=R$, show that

$$\int_C P(z)\,d\overline{z}=-2\pi i R^2 P'(a)\,.$$

(Here, for a continuous function $g(z)$ and a curve $\gamma: z=z(t)$, $a\le t\le b$, in $\mathbb{C}$ one defines $\int_C g(z)\,d\overline{z}$ by $\int_C g(z)\,d\overline{z} := \int_a^b g(z(t))\cdot\overline{z'(t)}\,dt$.)

Answers: 1. (a) $I=-\frac{2\pi i}{3}$. (b) $I=0$.