UPPSALA UNIVERSITET
MATEMATISKA INSTITUTIONEN
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Complex Analysis 10hp
Kand Ma2
Spring Term 2014

Home assignment 4

Note: Solutions to the problems should contain both detailed arguments and calculations (where appriopriate).

1. Find the principal part of

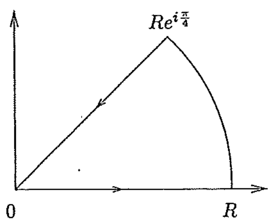
$$f(z) = \frac{1}{\sin z} + \frac{1}{\cos z - 1}$$

in the ring $\pi < |z| < 2\pi$.

2. Calculate the Fresnel integrals (optics)

$$\int_0^\infty \sin(x^2)\,dx \qquad \text{and} \qquad \int_0^\infty \cos(x^2)\,dx$$

by integrating the function e^{-z^2} over the contour

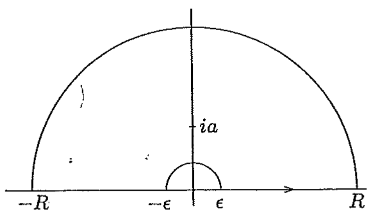


(Recall that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$, as proved in the course on Calculus of Several Real Variables.)

3. Calculate the integral

$$\int_0^\infty \frac{\log x}{x^2 + a^2}\,dx\,, \qquad (a > 0)\,.$$

(Hint: Use the contour



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4. Prove that all entire functions (i.e. analytic in $\mathbb{C}$) that are also injective take the form $f(z) = az + b$, with $a, b \in \mathbb{C}$ and $a \neq 0$.

[Hints: 1) Apply the Casorati-Weierstrass theorem to $f(\frac{1}{z})$.

2) You may have some use of the following result which we shall prove soon:

Thm: (Open mapping theorem) If f is analytic and non-constant in a region Ω, then f is an open mapping; that is, the image of any open set under f is open.]

Note: The home assignment 4 is to be left at latest on Friday, May 16th, at 5 p.m. in my mail-box.