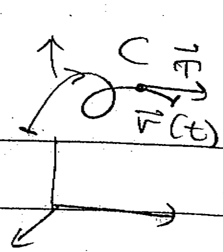
Kurvintegraler

C : $\vec{r}=\vec{r}(t)$, $a\le t\le b$



1) $f=f(x,y,z)$ – en funkt. på C

$$\int_C f\,ds = \int_a^b f(\vec{r}(t))\cdot|\vec{r}\,'(t)|\,dt$$

– oberoende av parametrisering och orientering

(Spec: längden $\ell(C)=\int_C 1\,ds = \int_a^b |\vec{r}\,'(t)|\,dt$)

2) $\vec{F}=\vec{F}(x,y,z)$ – ett v. fält på C

C – en orient. kurva

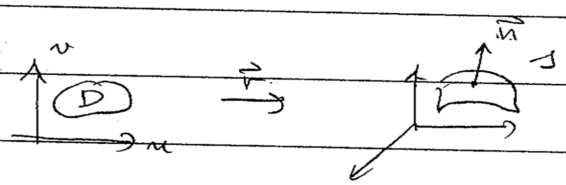
$$\int_C \vec{F}\cdot d\vec{r} = \int_C (\vec{F}\cdot\vec{T})\,ds = \int_C \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$$

– ober. av param.

– ber. av orientering

Ytintegraler :

S – en yta

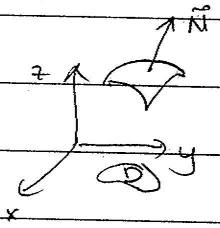


$|\vec{n}|=1$

1) $f=f(x,y,z)$ – en funktion på S

$$\iint_S f\,dS = \iint_D f(\vec{r}(u,v))\cdot|\vec{r}\,'_u\times\vec{r}\,'_v|\,du\,dv$$

(Spec: Area(S) = $\iint_S 1\,dS = \iint_D |\vec{r}\,'_u\times\vec{r}\,'_v|\,du\,dv$)



$$\iint_S f\,dS = \iint_D f\cdot\frac{|\vec{N}|}{|N_3|}\,dx\,dy$$

2) S – en orienterad yta

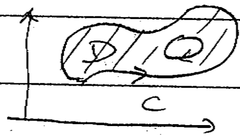
$\vec{F}=\vec{F}(x,y,z)$ – ett v. fält på S

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S (\vec{F}\cdot\vec{n})\,dS = \iint_D \vec{F}(\vec{r}(u,v))\cdot(\vec{r}\,'_u\times\vec{r}\,'_v)\,du\,dv$$

$$= \iint_D (\vec{F}\cdot\vec{N})\,\frac{1}{|N_3|}\,dx\,dy$$

Satser:

1) Greens Sats : i $\mathbb{R}^2$



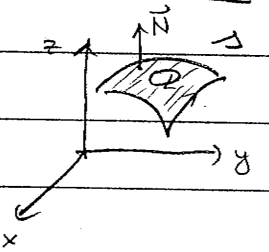
$C = \partial D$ - pos. orient.

$\vec{F}(x,y) = (P(x,y), Q(x,y))$ - ett C^1-v.fält på D

Sats (Green)

$$\boxed{\int_C \vec{F}\cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy}$$

2) Stokes' Sats : i $\mathbb{R}^3$

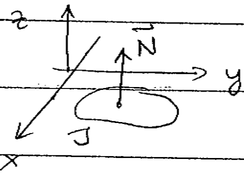


$\vec{F}(x,y,z)$ - v.fält på S

Sats (Stokes)

$$\boxed{\int_{\partial S} \vec{F}\cdot d\vec{r} = \iint_S \operatorname{curl}(\vec{F})\cdot d\vec{S}}$$

Obs: Om S ligger i xy-planet



Stokes = Green.

$$\left(\vec{F} = (P,Q,0) \Rightarrow \operatorname{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & 0 \end{vmatrix} = \left(x, x, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \right.$$

$\vec{N} = (0,0,1)$ $\quad S = D$

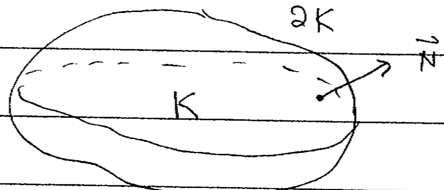
$$\Rightarrow \iint_S \operatorname{curl}(F)\cdot d\vec{S} = \iint_D \left(x, x, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\cdot(0,0,1)\frac{1}{|1|}d$$

$$\left. = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy \right)$$

3) Gauss' Sats : i $\mathbb{R}^3$

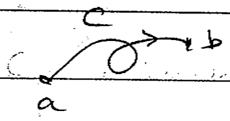
$\vec{F} = (F_1, F_2, F_3)$ - ett v.fält

$$\operatorname{div}(\vec{F}) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$



$$\boxed{\iint_{\partial K} \vec{F}\cdot d\vec{S} = \iiint_K \operatorname{div}(\vec{F})\, dV}$$

4. $\operatorname{curl}(\nabla \Phi) = \vec{0}$

5. $\int_C (\nabla \Phi) \cdot d\vec{r} = \Phi(b) - \Phi(a)$ om  $\Phi = \Phi(x, y, z)$ – en funktion

6. $\vec{F}$ – ett v. fält på $\Omega \subseteq \mathbb{R}^3$ – öppen

$\vec{F}$ – konservativt v. fält i Ω $\Leftrightarrow$ $\oint_\gamma \vec{F} \cdot d\vec{r} = 0$ för varje sluten kurva γ i Ω

7. (a) $\vec{F}$ – konservativt i Ω $\Rightarrow$ $\operatorname{curl}(\vec{F}) = \vec{0}$ i Ω (se 4)

(b) $\operatorname{curl}(\vec{F}) = \vec{0}$ i Ω och Ω – <u>enkelt sammanhängande</u>

$\Rightarrow$ $\vec{F}$ – konservativt i Ω.

(Om $\operatorname{curl}(\vec{F}) = \vec{0}$ i Ω men Ω – ej enkelt sammanhängande

$\Rightarrow$ vet ej, vidare arbete krävs.)