Formelsamling till kursen i Transformmetoder

Z-transformen

$a = (a_n)_0^\infty$	$A(z) = \mathcal{Z}[a](z) = \sum_{n=0}^{\infty} a_n z^{-n}$	
Allmänna regler		
$\alpha a_n + \beta b_n$	$\alpha A(z) + \beta B(z)$	(Z.1)
$\lambda^n a_n$	$A(\frac{z}{\lambda})$	(Z.2)
a_{n-k}, där $k \geq 1$ och $a_{-1} = \cdots = a_{-k} = 0$	$z^{-k} A(z)$	(Z.3)
a_{n+k}, där $k \geq 1$	$z^k A(z) - a_0 z^k - a_1 z^{k-1} - \cdots - a_{k-1} z$	(Z.4)
$n a_n$	$-z A'(z)$	(Z.5)
$(a * b)_n = \sum_{k=0}^{n} a_k b_{n-k}$	$A(z)B(z)$	(Z.6)
Speciella transformer		
$\begin{cases} 1 & \text{om } n = 0 \\ 0 & \text{om } n \geq 1 \end{cases}$	1	(Z.7)
1	$\frac{z}{z-1}$	(Z.8)
λ^n	$\frac{z}{z-\lambda}$	(Z.9)
n	$\frac{z}{(z-1)^2}$	(Z.10)
$n\lambda^n$	$\frac{\lambda z}{(z-\lambda)^2}$	(Z.11)
n^2	$\frac{z^2+z}{(z-1)^3}$	(Z.12)
$\binom{n}{k} \lambda^{n-k}$	$\frac{z}{(z-\lambda)^{k+1}}$	(Z.13)
$\cos \alpha n$	$\frac{z^2 - z\cos\alpha}{z^2 - 2z\cos\alpha + 1}$	(Z.14)
$\sin \alpha n$	$\frac{z \sin\alpha}{z^2 - 2z\cos\alpha + 1}$	(Z.15)

Laplacetransformen

$f(t)$	$\tilde{f}(s) = F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}\,dt$	
Allmänna regler		
$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$	(L.1)
$e^{at}f(t)$	$F(s-a)$	(L.2)
$f(at), \quad a > 0$	$\frac{1}{a}F(\frac{s}{a})$	(L.3)
$f(t-a)H(t-a), \quad a > 0$	$e^{-as}F(s)$	(L.4)
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	(L.5)
$f'(t)$	$sF(s) - f(0)$	(L.6)
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0)$ $-s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$	(L.7)
$\int_0^t f(u)\,du$	$s^{-1}F(s)$	(L.8)
$f * g(t) = \int_0^t f(u)g(t-u)\,du$	$F(s)G(s)$	(L.9)
Speciella funktioner		
$\delta(t)$	1	(L.10)
$H(t)$	$\frac{1}{s}$	(L.11)
$t^n, \quad n = 0, 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$	(L.12)
e^{at}	$\frac{1}{s-a}$	(L.13)
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(L.14)
$\cos at$	$\frac{s}{s^2+a^2}$	(L.15)
$\sin at$	$\frac{a}{s^2+a^2}$	(L.16)
$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$	(L.17)
$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$	(L.18)

Fouriertransformen

$f(t)$	$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$	
Allmänna regler		
$\alpha f(t) + \beta g(t)$	$\alpha\hat{f}(\omega) + \beta\hat{g}(\omega)$	(F.1)
$e^{i\alpha t}f(t)$	$\hat{f}(\omega - \alpha)$	(F.2)
$f(t - t_0)$	$e^{-it_0\omega}\hat{f}(\omega)$	(F.3)
$f(-t)$	$\hat{f}(-\omega)$	(F.4)
$f(at) \quad (a \neq 0)$	$\frac{1}{\lvert a\rvert}\hat{f}(\frac{\omega}{a})$	(F.5)
$tf(t)$	$i\,\frac{d\hat{f}}{d\omega}$	(F.6)
$f'(t)$	$i\omega\hat{f}(\omega)$	(F.7)
$\hat{f}(t)$	$2\pi f(-\omega)$	(F.8)
$f * g(t) = \int_{-\infty}^{\infty} f(u)g(t-u)\,du$	$\hat{f}(\omega)\hat{g}(\omega)$	(F.9)
Speciella funktioner		
$\chi_{[-a,a]}$	$\frac{2\sin a\omega}{\omega}$	(F.10)
$e^{-\lvert t\rvert}$	$\frac{2}{1+\omega^2}$	(F.11)
$\frac{1}{1+t^2}$	$\pi e^{-\lvert\omega\rvert}$	(F.12)
$e^{-t^2/2}$	$\sqrt{2\pi}e^{-\omega^2/2}$	(F.13)

Plancherels formler:

$$\int_{-\infty}^{\infty} |f(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\hat{f}(\omega)|^2\,d\omega$$

$$\int_{-\infty}^{\infty} f(t)\,\overline{g(t)}\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)\,\overline{\hat{g}(\omega)}\,d\omega$$

Fourierserier

Funktioner med period 2π

$$f(t) \sim \sum_{n=-\infty}^{\infty} c_n e^{int} = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt),$$

där

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)e^{-int}\,dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt, \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt$$

$$a_n = c_n + c_{-n}, \qquad b_n = i(c_n - c_{-n})$$

Parsevals formel:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(t)|^2\,dt = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{|a_0|^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(|a_n|^2 + |b_n|^2).$$

Funktioner med period T

Sätt $\Omega = 2\pi/T$

$$f(t) \sim \sum_{n=-\infty}^{\infty} c_n e^{in\Omega t} = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\Omega t + b_n \sin n\Omega t),$$

där

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-in\Omega t}\,dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\Omega t\,dt, \qquad b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\Omega t\,dt.$$

Parsevals formel:

$$\frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2\,dt = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{|a_0|^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(|a_n|^2 + |b_n|^2).$$

Några trigonometriska formler

$$2\sin a \sin b = \cos(a-b) - \cos(a+b)$$

$$2\sin a \cos b = \sin(a-b) + \sin(a+b)$$

$$2\cos a \cos b = \cos(a-b) + \cos(a+b)$$

$$2\sin^2 t = 1 - \cos 2t, \qquad 2\cos^2 t = 1 + \cos 2t$$