TIBETAN CALENDAR MATHEMATICS

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Abstract. The calculations of the Tibetan calendar are described, using modern mathematical notations instead of the traditional methods.

1. Introduction

The Tibetan calendar is derived from the Indian calendar tradition; it has the same general structure as Indian calendars, see [1], but the details differ significantly. The basis for the Tibetan calendar is the Kalacakra Tantra, which was translated from Sanskrit into Tibetan in the 11th century. (Traditional date of the translation is 1027 when the first 60 year cycle starts.) It is based on Indian astronomy, but much modified. The calendar became the standard in Tibet in the second half of the thirteenth century.

As in Indian calendars [1], months are lunar (from new moon to new moon), but numbered by the corresponding solar months, while days are numbered by the corresponding lunar days. Since these correspondences are not perfect, there are occasionally two months with the same number, in which case the first of them is regarded as a leap month, and occasionally an omitted date or two days with the same date (in this case I am not sure which one to regard as leap day, so I will call them earlier and later). Unlike modern Indian calendars, there are no omitted months.

Various improvements of the calculations has been suggested over the centuries, see [4] and [3, Chapter VI], and different traditions follow different rules for the details of the calculation. There are at least two versions (*Phugpa* and *Tsurphu*) of the Tibetan calendar in use today by different groups inside and outside Tibet, see Appendix B. The different versions frequently differ by a day or a month.

The description below refers to the Phugpa tradition, introduced 1447, which is followed by the Dalai Lama and is the most common version; it can be regarded as the standard version of the Tibetan calendar. The differences in the Tsurphu tradition are discussed in Appendix B, but some details remain obscure to me.

I will separate the discussion of the Tibetan calendar into two main parts. In the first part (Sections 4–6), the months are regarded as units and I discuss how they are numbered, which implies the partitioning of them into years and also shows which months that are leap months. In the second part

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(Sections 7–10), I discuss the coupling between months and days, including finding the actual days when a month begins and ends and the numbering of the days. Finally, some further calculations are described (Section 11) and some mathematical consequences are given (Sections 13–14). Calculations for the planets and other atrological calculations are described in the appendices.

The description is based mainly on the books by Schuh [4] and Henning [3], but the analysis and mathematical formulations are often my own. For further study I recommend in particular the detailed recent book by Henning [3], which contains much more material than this paper.

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2. Notation

Mixed radix notation. Traditional Tibetan calculations are made with rational numbers expressed in a positional system with mixed radices. I usually use standard notation for rational numbers, but when comparing them with the traditional expressions, I use notations of the type (of varying length)

$$a_0; a_1, a_2, a_3 \ (b_1, b_2, b_3) \qquad \text{meaning} \qquad a_0 + \frac{a_1 + (a_2 + (a_3/b_3))/b_2}{b_1}.$$

Formally, we have the inductive definition

$$a_0; a_1, a_2, \dots, a_n \ (b_1, b_2, \dots, b_n) := a_0 + \frac{a_1; a_2, \dots, a_n \ (b_2, \dots, b_n)}{b_1}$$

for $n \geq 2$, and $a_0; a_1 \ (b_1) := a_0 + a_1/b_1$. We will usually omit a leading 0. (This notation is taken from Henning [3], although he usually omits all or most of the radices since they are given by the context. Schuh [4] uses the similar notation $[a_1, a_2, \dots, a_n]/(b_1, b_2, \dots, b_n)$ meaning either $0; a_1, a_2, \dots, a_n \ (b_1, b_2, \dots, b_n)$ or $a_1; a_2, \dots, a_n \ (b_2, \dots, b_n)$.)

Angular units. It will be convenient, although somewhat unconventional, to express longitudes and other angular measurements in units of full circles. To obtain values in degrees, the numbers should thus be multiplied by 360. (A Tibetan astrologer would probably prefer multiplying by 27 to obtain lunar mansions (= lunar station = *naksatra*) and fractions thereof. A Western astrologer might prefer multiplying by 12 to obtain values in signs. A mathematician might prefer multiplying by 2π (radians).)

For angular measurements, full circles are often to be ignored (but see below); this means with our convention that the numbers are considered modulo 1, i.e., that only the fractional part matters.

Boolean variables. For a Boolean variable ℓ, i.e. a variable taking one of the values *true* and *false*, we use $\ell := \{\mathcal{P}\}$ to denote that $\ell = true$ if and only if the condition $\mathcal{P}$ holds; we further let $[\ell]$ be the number defined by $[\ell] = 1$ when $\ell = true$ and $[\ell] = 0$ when $\ell = false$.

Julian day number. The *Julian day number* (which we abbreviate by JD) for a given day is the number of days that have elapsed since the epoch 1 January 4713 BC (Julian); for days before the epoch (which hardly concern the Tibetan calendar), negative numbers are used. The Julian day numbers thus form a continuous numbering of all days by $\dots, -1, 0, 1, 2, \dots$. Such a numbering is very convenient for many purposes, including conversions between calendars. The choice of epoch for the day numbers is arbitrary and for most purposes unimportant; the conventional date 1 January 4713 BC (-4712 with astronomical numbering of years) was originally chosen by Scalinger in 1583 as the origin of the Julian period, a (cyclic) numbering of years, and was later adapted into a numbering of days. See further [2, Section 12.7].

Dershowitz and Reingold [1] use another day number, denoted by RD, with another epoch: RD 1 is 1 January 1 (Gregorian) which is JD 1721426. Consequently, the two day numbers are related by JD = RD + 1721425.

The Julian day number was further developed into the *Julian date*, which is a real number that defines the time of a particular instance. The fractional part of the Julian date shows the fraction of a day that has elapsed since noon GMT (UT); thus, if n is an integer, then the Julian date is $n.0$ (i.e., exactly n) at noon GMT on the day with Julian day number n.

It is important to distinguish between the Julian day number and the Julian date, even if they are closely related. Both are extremely useful, but for different purposes, and much confusion has been caused by mixing them. (We follow [2] in using slightly different names for them, but that is not always done by other authors.) For this study, and most other work on calendars, the Julian day number is the important concept. Note that the Julian day number is an integer, while the Julian date is a real number. (A computer scientist would say that they have different types.) Moreover, the Julian day number numbers the days regardless of when they begin and end, while the Julian date depends on the time of day, at Greenwich. Hence, to convert a Julian date to a Julian day number, we need in practice know both the local time the day begins and the time zone, while these are irrelevant for calculations with the Julian day number. For example, 1 January 2007 has JD 2454102, everywhere. Thus the Julian date 2454102.0 is 1 January 2007, noon GMT (UT), and the new year began at Julian date 2454101.5 in Britain, but at other Julian dates in other time zones. The Tibetan day begins at dawn, about 5 am local time (see Remark 6 below), but we do not have to find the Julian date of that instance.

Other notations. We let $\bmod$ denote the binary operation defined by $m \bmod n = x$ if $x \equiv m \pmod{n}$ and $0 \le x < n$ (we only consider $n > 0$, but m may be of any sign; care has to be taken with this in computer implementations).

Similarly (following [1]), we let amod denote the binary operation defined by $m \ \mathrm{amod}\ n = x$ if $x \equiv m \pmod n$ and $0 < x \leq n$. This means that $m \ \mathrm{amod}\ n = m \mod n$ except when m is a multiple of n; then $m \mod n = 0$ and $m \ \mathrm{amod}\ n = n$. For integers m and n (the usual case, and the only one used in this paper), $m \ \mathrm{amod}\ n = 1 + (m-1) \mod n$.

We use the standard notations $\lfloor x \rfloor$ and $\lceil x \rceil$ for the integers satisfying $x - 1 < \lfloor x \rfloor \leq x$ and $x \leq \lceil x \rceil < x + 1$, i.e. x rounded down and up to an integer, and $\mathrm{frac}(x) := x - \lfloor x \rfloor = x \mod 1$ for the fractional part. (Again, care has to be taken for $x < 0$ in computer implementations.)

3. Numbering and naming of years

Several ways of numbering or naming Tibetan years are used. One common method, especially among westeners, is to simply number a Tibetan year by the Gregorian (or Julian) year in which it starts (and where most of it falls). For convenience, we will use this numbering below.

Another method is to number the years from an era 127 BC (the traditional ascent of the first Tibetan king); the Tibetan year starting in Gregorian year Y will then be numbered $Y + 127$. Both methods are used by Tibetans; the Tibetan calendar [5] has titles in both Tibetan and English; with 2003 in the English title and 2130 in the Tibetan.

Moreover, and more importantly, each year is named according to a 60 year cycle. Actually, there are two different 60 year cycles of names, one Indian and one Chinese. Of course, since the cycles have the same length, there is a 1–1 correspondence between the names. When naming years, the Chinese cycle is almost always used, and sometimes the Indian and Chinese cycle names are used together. (For example, the Chinese cycle names are used in the titles of the calendars [5] and [6].)

The Indian cycle is a list of 60 different names, in Sanskrit or Tibetan, see Appendix A. The cycle is named after its first year as Prabhava (*rab byung*). The cycles are numbered, with the first cycle beginning in AD 1027, which means that each year can be unambiguously identified by its name in the cycle and the number of the cycle.

Year n in cycle m (with $1 \leq n \leq 60$ and, presumably, $m \geq 1$) thus corresponds to Gregorian or Julian year Y given by

$$Y = 1027 + (m-1)60 + (n-1) = 60m + n + 966. \tag{1}$$

Conversely,

$$n = (Y - 1026) \ \mathrm{amod}\ 60 = (Y - 6) \ \mathrm{amod}\ 60, \tag{2}$$

$$m = \left\lceil \frac{Y - 1026}{60} \right\rceil. \tag{3}$$

For example, AD 2007 is the 21st year in the 17th Prabhava cycle, which began in 1987.

The Chinese cycle is identical to the one used in the Chinese calendar [1]. The cycles start 3 years before the Indian ones, so the first year (Prabhava, *rab byung*) in the Indian cycle is the fourth year (Fire–female–Rabbit) in the Chinese cycle. The full correspondence is given in Appendix A. The last cycle thus started 1984. Hence, or by (2), year Y is number $(Y-3) \operatorname{amod} 60$ in the cycle. The Chinese cycles are not numbered in the Tibetan Calendar.

The Chinese 60 year cycle is a combination of two cycles, of 10 and 12 years respectively. (Note that the least common multiple of 10 and 12 is 60; since 10 and 12 have greatest common divisor 2, only half of the 10×12 combinations are possible.)

The 10 year cycle consists in China of 10 different names (proper names with no other English translation) called celestial stems. Each celestial stem is associated with an element (wood, fire, earth, iron, water) and a quality (female or male), see Table 1. (In Chinese, the two qualities are the well-known *yin* and *yang*.) Note that the 2 qualities are alternating and thus are given by the year mod 2, while the 5 elements are repeated 2 years each in the 10 year cycle. As a consequence, each celestial stem corresponds to a unique (element, quality) pair, and in the Tibetan calendar, the element and quality are used to name the year; the celestial stems are usually not used.

Using the numbering in Table 9, year Y is $z := (Y-3) \operatorname{amod} 10$ in the Chinese 10 year cycle, and has element $\lceil z/2 \rceil$.

The 12 year cycle is the well-known cycle of animals found in the Chinese and many other Asian calendars, see Table 2.

The Tibetan name for a year according to the Chinese cycle is thus given as Element–Quality–Animal. Note that the quality, being the year mod 2, also is determined by the animal (since 2 divides 12), as shown in Table 2. Indeed, the quality is often omitted and only Element–Animal is used as the name of the year. For example, AD 2007 is the 24th year in the Chinese cycle (21st in the Indian) and is thus Fire–female–Pig or simple Fire–Pig.

The civil year stars, unsurprisingly, with month 1; note that in case there is a leap month 1, the year begins with the leap month, which precedes the regular month 1. (This happened in 2000. See also Remark 15.) The first day of the year is also celebrated as a major holiday (*Losar*; the Tibetan New Year).

Remark 1. Most Tibetan calendars start, however, with month 3; this month is identified with the Kalacakra month *Caitra*, which was considered the beginning of the Kalacakra year [3, p. 194] (as in most Indian calendars [1]). The standard numbering system is known as *Mongolian month* (*hor zla*), and was introduced in the 13th century when the Tibetan new year was moved to be the same as the Mongolian [3, p. 145].

year		element	quality	celestial stem (Chinese)
1	8	wood	male	jiǎ
2	9	wood	female	yǐ
3	10	fire	male	bǐng
4	1	fire	female	dīng
5	2	earth	male	wù
6	3	earth	female	jǐ
7	4	iron	male	gēng
8	5	iron	female	xīn
9	6	water	male	rén
10	7	water	female	guǐ

TABLE 1. The 10 year cycle. The first number on each line shows the year mod 10 counted from the start of a Chinese cycle; the second shows the year mod 10 counted from the start of a Prabhava cycle.

year		animal	quality
1	10	Mouse	male
2	11	Ox	female
3	12	Tiger	male
4	1	Rabbit	female
5	2	Dragon	male
6	3	Snake	female
7	4	Horse	male
8	5	Sheep	female
9	6	Monkey	male
10	7	Bird	female
11	8	Dog	male
12	9	Pig	female

TABLE 2. The 12 year cycle. The first number on each line shows the year mod 12 counted from the start of a Chinese cycle; the second shows the year mod 12 counted from the start of a Prabhava cycle.

4. Numbering of months; general theory

The months are described by year, number (from 1 to 12) and, possibly, the label "leap". We can thus think of each month as labelled by a triple (Y, M, ℓ), where $Y \in \mathbb{Z}$, $m \in \{1, \dots, 12\}$ and $\ell \in \{\mathit{true}, \mathit{false}\}$ is a Boolean variable.

Let us, for the time being, use the beginning of an arbitrary month M_0 year Y_0 as the epoch.

We will use a consecutive numbering of all months, starting with 0 (for convenience) at the epoch. To avoid confusion with the yearly numbering from 1 to 12, we use the term *month count* and the variable n for this numbering. Thus, month count 0 is month M_0 of year Y_0. (The month count is often called the *true month*, but we will use the term true month for a rational number defined in Section 6, see (34). The month count equals the true month rounded to an integer according to the discussion after (37), but we will use different names to avoid confusion.)

The goal is to find the correspondence (mapping) between n and (Y, M, ℓ).

The key is the mean solar longitude (MSL). This increases by the same amount, say s_1, for each month. Hence MSL at the beginning of month count n, which I denote by $\mathrm{MSL}(n)$, is given by the linear formula

$$\mathrm{MSL}(n) = s_1 n + s_0, \tag{4}$$

where $s_0 = \mathrm{MSL}(0)$ is the MSL at the epoch. (Thus, s_1 and s_0 are constants in the system; s_0 depends on the choice of epoch but s_1 does not.) The Tibetan constant s_1 equals $\frac{65}{804} = \frac{65}{67 \cdot 12}$, see Section 5.

Moreover, the zodiac contains 12 evenly spaced *definition points* (to use the terminology by Henning [3]) (*sgang*), one in each sign. Let us denote these (and their longitudes) by $p_1, \dots, p_{12}$. According to Henning [3], the rule is:

> *A month where the mean sun passes a definition point p_M is given number M.*

Since $s_1 < \frac{1}{12}$, the spacing of the definition points, the mean sun can never pass two definition points in one month, but sometimes it does not pass any of them; in that case the month is designated as a leap month, and is given the number of the next month. In both cases, the number of the month is thus given by the first definition point p_M coming after $\mathrm{MSL}(n)$. (We do not have to worry about the exact definition in the ambiguous case when $\mathrm{MSL}(n)$ exactly equals some p_M; the constants in the Phugpa system are such that this never will happen, see below. I have therefore just chosen one version in the formulas below.)

Remark 2. The rule above for numbering the months is the same as in many Indian calendars [1, Chapters 9 and 17], and similar to the rule in the Chinese calendar [1, Chapters 16]; in these calendars, the definition points are beginnings of the zodiacal signs, i.e., multiples of $30° = 1/12$. (In the Chinese calendar, the definition points are called *(major) solar terms*. In most (but not all) Indian calendars, month 1 (New Year) is defined by the vernal equinox, i.e., $p_1 = 0$ in our notation; the exact rule in the Chinese calendar is that the winter solstice ($270° = 3/4$) occurs in month 11, which corresponds to $p_{11} = 3/4$, but the details differ a little.) Note however, that the (present) Chinese and Indian calendars use the true motion of sun and moon, while the Tibetan uses the mean motion, leading to regularly spaced leap months in the Tibetan calendar, but not in the Chinese and Indian ones. (It also leads to skipped months sometimes in the Indian calendars.) Note further that the numbering of leap months differ in the Chinese calendar, where a leap month is given the number of the preceding month.

The longitude in (4) is naturally taken modulo 1, i.e., considering only the fractional part. But a moment's consideration shows that the integer part shows the number of elapsed full circles of the sun, i.e., the number of years. We thus use $\mathrm{MSL}(n)$ for the real number defined by (4) and can write the rules defining (Y, M, ℓ) as follows, with $p_0 := p_{12} - 1$,

$$Y - Y_0 + p_{M-1} < \mathrm{MSL}(n) \leq Y - Y_0 + p_M, \tag{5}$$

$$\ell = [\mathrm{MSL}(n+1) \leq Y - Y_0 + p_M]. \tag{6}$$

Furthermore, the points p_M are evenly spaced, so $p_M = p_0 + M/12$.

Let us for simplicity write $Y' := Y - Y_0$. Then (5) can be rewritten

$$Y' + \frac{M-1}{12} < \mathrm{MSL}(n) - p_0 \leq Y' + \frac{M}{12} \tag{7}$$

or

$$12Y' + M - 1 < 12(\mathrm{MSL}(n) - p_0) \leq 12Y' + M. \tag{8}$$

Hence, if we use (4) and further define

$$\alpha := 12(s_0 - p_0), \tag{9}$$

we have

$$12Y' + M = \lceil 12(\mathrm{MSL}(n) - p_0) \rceil = \lceil 12 s_1 n + \alpha \rceil. \tag{10}$$

Consequently, we can calculate (Y, M) from n by

$$x := \lceil 12 s_1 n + \alpha \rceil, \tag{11}$$

$$M := x \ \text{ amod } \ 12, \tag{12}$$

$$Y := \frac{x - M}{12} + Y_0. \tag{13}$$

To complete the calculations of (Y, M, ℓ) from n, we find similarly from (6), or simpler by (10) because a month is leap if and only if it gets the same number as the following one,

$$\ell := \big\{ \lceil 12 s_1 (n+1) + \alpha \rceil = \lceil 12 s_1 n + \alpha \rceil \big\}. \tag{14}$$

Remark 3. To verify the assertion that $\mathrm{MSL}(n)$ never equals some definition point, note that the calculations above show that this would happen if and only if $12 s_1 n + \alpha$ would be an integer. Since $12 s_1 = \frac{65}{67}$, this can happen only if 67α is an integer (and in that case it would happen for some n); we will see in (24) below that this is not the case.

Conversely, given (Y, M, ℓ), we can find n from (7), noting that if there are two possible values of n, then we should choose the smaller one if $\ell = \textit{true}$ and the larger one if $\ell = \textit{false}$. If $\ell = \textit{false}$, then (7) and (4) thus show that n is the largest integer such that

$$s_1 n + s_0 - p_0 \leq Y' + \frac{M}{12}$$

or, recalling (9),

$$s_1 n \leq Y' + \frac{M-\alpha}{12};$$

if $\ell = \mathit{true}$, this value of n should be decreased by 1. Hence, in all cases, n can be computed from (Y, M, ℓ) by

$$n = \left\lfloor \frac{12(Y - Y_0) + M - \alpha}{12 s_1} \right\rfloor - [\ell]. \tag{15}$$

An alternative formula, which has the advantage that if there is no leap month M in year Y, then (Y, M, true) gives the same result as (Y, M, false), is given by

$$n = \left\lfloor \frac{12(Y - Y_0) + M - \alpha - (1 - 12 s_1)[\ell]}{12 s_1} \right\rfloor; \tag{16}$$

to see this, note that if $\ell = \mathit{false}$, then (16) and (15) give the same result, while if $\ell = \mathit{true}$, then (16) gives 1 more than (15) applied to $(Y, M-1, \mathit{false})$, which is the month preceding leap month M (also if $M = 1$, when this really is $(Y-1, 12, \mathit{false})$).

5. The Phugpa constants for months

The increase s_1 of the MSL per month is traditionally given in mixed radices as $2; 10, 58, 1, 17\ (60, 60, 6, 67)$ (lunar) mansions, i.e., in full circles,

$$s_1 = 2, 10, 58, 1, 17\ (27, 60, 60, 6, 67) = \frac{65}{804} = \frac{65}{67} \cdot \frac{1}{12}. \tag{17}$$

(This can also be expressed as 65/67 signs, or $29\frac{7}{67}^{\circ}$.)

Remark 4. In other words, 804 (lunar) months = 65 (solar) years. If we define a solar month as 1/12 of a year, we can also express this as

$$67 \text{ (lunar) months} = 65 \text{ solar months}. \tag{18}$$

This is a fundamental relation in the Tibetan calendar following the Kalacakra Tantra.

The first definition point p_1 is according to Henning [3] $23; 6\ (60)$ mansions, i.e.

$$p_1 = 23, 6\ (27, 60) = \frac{77}{90} \tag{19}$$

and thus

$$p_0 = p_1 - \frac{1}{12} = \frac{139}{180}. \tag{20}$$

If we, for our convenience, number a Tibetan year by the Gregorian (or Julian) year in which it starts, then Schuh [4] uses the epoch

$$\begin{aligned} y_0 &= 806, \\ s_0 &= 24, 57, 5, 2, 16\ (27, 60, 60, 6, 67) = \frac{743}{804} \end{aligned} \tag{21}$$

while Henning [3] uses the epoch

$$\begin{aligned} y_0 &= 1927, \\ s_0 &= 25, 9, 10, 4, 32 \ \ (27, 60, 60, 6, 67) = \frac{749}{804} \end{aligned} \tag{22}$$

and Lai and Dolma [8] use the epoch

$$\begin{aligned} y_0 &= 1987, \\ s_0 &= 1. \end{aligned} \tag{23}$$

Thus the years Y_0 differ by 1121 between the first two epochs and the values of s_0 differ by $\frac{6}{804} = \frac{1}{134}$. Since $1121 + \frac{1}{134} = 13866 s_1$, the two epochs give the same result in (4), and thus in all formulas above, if we let the values of n differ by 13866. In other words, there are 13866 months between the two epochs, and they are equivalent and give the same results, except that the month counts differ by 13866.

The third epoch differs form the second by 60 further years and the values of s_0 differ by $\frac{55}{804}$. Since $60 + \frac{55}{804} = 743 s_1$, the two epochs give the same result in (4), and thus in all formulas above, if we let the values of n differ by 743. In other words, all third epochs give the same results, with the month count for the third epoch shifted by 743 from the second and 14609 from the first one.

We will use the labels E806, E1927 and E1987 to distinguish the three epochs. The epoch E806 (several centuries before the calendar came to Tibet) is the traditional one from Kalacakra Tantra, but later epochs are convenient for hand calculations because they give smaller numbers. (It is convenient and customary to use the start of the current 60-year cycle as the epoch.)

The constant α defined by (9) equals

$$\alpha = 12(s_0 - p_0) = \frac{1832}{1005} = 1 + \frac{827}{1005} \qquad \text{(E806);} \tag{24}$$

or

$$\alpha = 12(s_0 - p_0) = \frac{1922}{1005} = 1 + \frac{917}{1005} \qquad \text{(E1927);} \tag{25}$$

$$\alpha = 12(s_0 - p_0) = \frac{41}{15} = 1 + \frac{2}{15} \qquad \text{(E1987).} \tag{26}$$

Remark 5. For both E806 and E1927, we have $1 < \alpha < 2$, and thus (10) shows that month count 0 has $m = 2$, so it is the second month of year Y_0 (there is no leap month in the beginning of Y_0). Equivalently, and more in line with traditional descriptions, month count 1 is the third month (Sanskrit *Caitra*) of Y_0. (*Caitra* is the first month in most Indian calendars.)

The epoch E1987 has $2 < \alpha < 3$, and thus month count 0 has $m = 3$; hence in this case we count months *after* the third month of Y_0.

6. Leap months

Since the months are numbered by the corresponding solar months, it follows from (18) that there are 2 leap months in 67 months, and the linear rules above imply that the leap months are spaced regularly, with a distance alternating between 33 and 34 months. It follows that the leap months have a cycle of 65 years. In this cycle there are $12 \cdot 65 = 780$ regular months and 24 leap months. Since 67 and 12 are coprime, the leap months are evenly distributed over the year, so in each cycle there are exactly two leap months with each number $1, \dots, 12$.

Let us write $M' := 12(Y - Y_0) + M$; this can be interpreted as a solar month count (or we can interpret month M year Y as month M' year Y_0). Since $12s_1 = 65/67$, we can write (15) as

$$n = \left\lfloor \frac{67}{65}(M' - \alpha) \right\rfloor - [\ell] = \left\lfloor \frac{67M'}{65} - \frac{67\alpha}{65} \right\rfloor - [\ell], \tag{27}$$

and since $\frac{67}{65}M'$ is an integer multiple of $1/65$, this value is not affected if 67α is replaced by the integer $\lceil 67\alpha \rceil$. We thus define

$$\beta := \lceil 67\alpha \rceil \tag{28}$$

and have

$$n = \left\lfloor \frac{67M' - \beta}{65} \right\rfloor - [\ell] = M' + \left\lfloor \frac{2M' - \beta}{65} \right\rfloor - [\ell]. \tag{29}$$

There is a leap month M in year Y if and only if the month count for (Y, M, false) jumps by 2 from the preceding regular month $(Y, M-1, \mathit{false})$. By (29), this happens exactly when $2M' - \beta$ just has passed a multiple of 65, i.e., when $2M' - \beta \equiv 0$ or $1 \pmod{65}$. Thus:

There is a leap month M in year Y if and only if

$$2M' \equiv \beta \textit{ or } \beta + 1 \pmod{65},$$

where $M' := 12(Y - Y_0) + M$.

For our three epochs we have

$$\beta = 123 \equiv 58 \pmod{65} \qquad \text{(E806)}; \tag{30}$$

$$\beta = 129 \equiv 64 \pmod{65} \qquad \text{(E1927)}; \tag{31}$$

$$\beta = 184 \equiv 54 \pmod{65} \qquad \text{(E1987)}. \tag{32}$$

The traditional version of this rule is stated slightly differently, for example by Henning [3]. First, (solar) months are counted starting after month 3 (*Caitra*); hence the "number of zodiacal months" (*khyim zla*) is $M' - 3$. Secondly, after multiplying this number $M' - 3$ by $67/65$, one adds $\beta^*/65$, where the constant β^* is given by

$$\beta^* := (54 - \beta) \bmod 65; \tag{33}$$

thus, by (30)–(32), $\beta^* = 61$ for E806, $\beta^* = 55$ for E1927 and $\beta^* = 0$ for E1987. The result is called the *true month* (*zla dag*); furthermore, if we write the fractional part as $ix/65$, the integer ix is called the *intercalation index* [3] (*zla phro* [8]). Thus the true month is,

$$\frac{67(M'-3)}{65}+\frac{\beta^*}{65} \tag{34}$$

and

$$ix := 67(M'-3)+\beta^* \bmod 65 = 2(M'-3)+\beta^* \bmod 65 \tag{35}$$
$$\equiv 2M'+\beta^*-6 \equiv 2M'-\beta+48 \pmod{65}. \tag{36}$$

The rule above is thus equivalent to:

> *There is a leap month M in year Y if and only if the intercalation index $ix \equiv 48$ or $49 \pmod{65}$.*

This is the traditional Phugpa form of the leap month rule.

Furthermore, $3\cdot 67-\beta \equiv 71-\beta \equiv \beta^*+17 \pmod{65}$, and thus (29) can be written

$$n = \left\lfloor \frac{67(M'-3)+\beta^*}{65}+\frac{17}{65}\right\rfloor - [\ell], \tag{37}$$

provided we use the correct starting point for the month count ($n=1$ for month 3 or 4 depending on whether $\beta^* \geq 48$ or not, cf. Remark 5). This implies that for a regular month, the month count n is obtained from the true month (34) by rounding down to the nearest integer if $ix < 48$ and rounding up if $ix \geq 48$; for a leap month (when $ix = 48$ or 49), we always round down. This is the traditional Phugpa rule to calculate the month count (given by Henning [3] in somewhat different words).

7. Days

Each (lunar) month is (as in the Indian calendars) divided into 30 lunar days (*tshes zhag,* Sanskrit *tithi*); these thus do not correspond exactly to the calendar days. During each of these lunar days, the elongation of the moon thus increases by $1/30$ ($= 12°$).

The calendar computations, unlike the Indian ones, do not include a function calculating the elongation at a given time; instead the computations use the inverse function, giving the time when the elongation has a given value. We denote this function by $true\_date(d,n)$, giving the date at the end of the lunar day d in month count n. The value of this function is a real (rational) number; traditionally it is counted modulo 7, and the integer part yields the day of week, but we will treat it as a real number so that the integer part directly gives the Julian day number JD of the calendar day. A different constant m_0 below will give RD [1] instead. (The fractional part has, I think, astrological use, but can be ignored for our purpose.)

The basic rule is:

> *A calendar day is labelled by the lunar day that is current at the beginning of the calendar day.*

In other words, a lunar day gives its name (number and month) to the calendar day where the lunar day ends. (Thus we obtain the JD of the calendar day by taking the floor of *true_date* at the end of the lunar day.) There are two special cases covered by the rule above: if two lunar days end the same calendar day, the calendar day gets the name of the first of them; if no lunar day ends a given calendar day, then that day gets the same name as the following day. See further Section 9.

Remark 6. We do not have to worry about when the day starts; the calendar day is from dawn to dawn, but the formulas take this into account (at least theoretically), and no further modification is done. In particular, no calculation of sunrise is required (as it is in Indian calendar calculations [1]). Thus, the *true_date* should be regarded as a kind of local Julian date that is offset from the standard astronomical one, which assumes integer values at noon UT (=GMT), so that it instead assume integer values at local (mean) dawn. (Henning [3, pp. 10–11] specifies the start of the day as mean daybreak = 5 am local mean solar time.)

The *true_date* is calculated by first calculating a simpler *mean_date*, corresponding to the linear mean motion of the moon, and then adjusting it by the equations of the moon and sun, which are determined by the anomalies af the moon and sun together with tables. (The tables are really approximation to sine, suitably scaled. A similar table is used for each planet; no general sine table is used; another difference from Indian calendars.) The anomalies, in turn, are also calculated by linear functions.

Traditional hand calculations calculate the mean quantities first for the beginning of the month, i.e. the end of the preceding month. (This corresponds to taking day $d = 0$ below. Note that this usually gives a time during the last calendar day of the preceding calendar month.) Then the quantities are adjusted to give the values for a given day. We will combine the two steps into one, giving the values for month count n and day d. (Lai and Dolma [8] refer to tables rather than doing multiplications to obtain the adjustments for d for *mean_date* and (without explanation) for *mean_sun*, but the results are the same.)

Remark 7. The traditional hand calculations express the various quantities as sequences (columns) of integers, which should be interpreted as rational numbers in mixed radices; the radices are fixed but different sequences are used for the different functions. We will not use this, except for expressing some constants (for comparison with other sources). For explanations and examples of the way the traditional hand calculations are performed, see Henning [3], Schuh [4], Lai and Dolma [8].

8. Astronomical functions

The Phugpa tradition uses the following functions. As said above, these give the values at the end of lunar day d in month count n. I use below E806, but give also the corresponding constants for E1927 and E1987.

The mean date (*gza' bar pa*) is

$$mean\_date(d,n) := n \cdot m_1 + d \cdot m_2 + m_0, \tag{38}$$

where

$$m_1 := 29; 31, 50, 0, 480 \ \ (60, 60, 6, 707) = \frac{167025}{5656} \quad (\approx 29.530587), \tag{39}$$

$$m_2 := 0; 59, 3, 4, 16 \ \ (60, 60, 6, 707) = \frac{11135}{11312} \quad \Big(= \frac{m_1}{30}\Big), \tag{40}$$

$$m_0 := 0; 50, 44, 2, 38 \ \ (60, 60, 6, 707) + 2015501 = 2015501 + \frac{4783}{5656}. \tag{41}$$

Remark 8. The traditional reckoning counts days modulo 7 only, i.e. day of week (see Section 10); this is of course enough to construct a calendar month by month. My version gives the JD directly. (To be precise, the traditional result is obtained by adding 2 to the value above before taking the remainder modulo 7, because $2015501 \equiv -2 \pmod 7$; cf. (53).)

The traditional value is therefore $m_1 := 1; 31, 50, 0, 480 \ (60, 60, 6, 707)$, or rather $m_1 := 1, 31, 50, 0, 480 \ (7, 60, 60, 6, 707)$ (the denominator 7 means that we regard the numbers as fractions of weeks, thus ignoring integer parts). Similarly, the constant 2015501 in m_0 (to get the result in JD) is my addition and not traditional. (To get RD, use 294076 instead.)

Remark 9. For E1927 [3], with the base for n differing by 13866, m_1 and m_2 are the same but m_0 is instead

$$\begin{aligned} m_0 + 13866 \cdot m_1 &= 2424972 + \frac{5457}{5656} \\ &= 2015501 + 409471 + \frac{5457}{5656} \\ &= 2015501 + 409465 + 6; 57, 53, 2, 20 \ \ (60, 60, 6, 707), \end{aligned}$$

traditionally written as $6; 57, 53, 2, 20 \ (60, 60, 6, 707)$. (Note that $409465 = 7 \cdot 58495$ a multiple of 7, i.e. a whole number of weeks; hence the this gives the correct day of week.)

Similarly, for E1987 [8], with the base for n differing by 14609, m_1 and m_2 are the same but m_0 is instead

$$\begin{aligned} m_0 + 14609 \cdot m_1 &= 2446914 + \frac{135}{707} \\ &= 2015501 + 431413 + \frac{135}{707} \\ &= 2015501 + 431410 + 3; 11, 27, 2, 332 \ \ (60, 60, 6, 707), \end{aligned}$$

traditionally written as $3; 11, 27, 2, 332 \ (60, 60, 6, 707)$.

Similarly, the mean longitude of the sun (*nyi ma bar pa*) is

$$mean\_sun(d,n) = n \cdot s_1 + d \cdot s_2 + s_0, \tag{42}$$

where, cf. (17),

$$s_1 := 2,10,58,1,17\ \ (27,60,60,6,67) = \frac{65}{804} \quad \Big(= \frac{65}{12\cdot 67}\Big), \tag{43}$$

$$s_2 := 0,4,21,5,43\ \ (27,60,60,6,67) = \frac{13}{4824} \quad \Big(= \frac{s_1}{30}\Big), \tag{44}$$

$$s_0 := 24,57,5,2,16\ \ (27,60,60,6,67) = \frac{743}{804}. \tag{45}$$

Note that the function MSL(n) in (4) equals $mean\_sun(0,n)$, except that we interpret $mean\_sun$ modulo 1 since only the fractional part matters in the present section.

Remark 10. For E1927 [3], with the base for n differing by 13866, s_0 is instead (modulo 1, since only the fractional part matters)

$$s_0 + 13866\cdot s_1 \equiv 25,9,10,4,32\ \ (27,60,60,6,67) = \frac{749}{804}.$$

For E1987 [8], with the base for n differing by 14609, s_0 is instead

$$s_0 + 14609\cdot s_1 \equiv 0.$$

This vanishing of the initial value, which recurs every 65th year (804th month), is called *nyi ma stong bzhugs* "sun empty enter" [8]. (s_0 was given as 1 in (23), but for the purpose of the present section only the fractional part matters.)

Thirdly, the anomaly of the moon (*ril po*) is

$$anomaly\_moon(d,n) := n\cdot a_1 + d\cdot a_2 + a_0,$$

where (but see Remark 14 below for an alternative for a_2)

$$a_1 := 2,1\ \ (28,126) = \frac{253}{3528}, \tag{46}$$

$$a_2 := 1,0\ \ (28,126) = \frac{1}{28}, \tag{47}$$

$$a_0 := 3,97\ \ (28,126) = \frac{475}{3528}. \tag{48}$$

Remark 11. For E1927 [3], with the base for n differing by 13866, a_0 is instead (modulo 1, since only the fractional part matters)

$$a_0 + 13866\cdot a_1 \equiv 13,103\ \ (28,126) = \frac{1741}{3528}.$$

For E1987 [8], with the base for n differing by 14609, a_0 is instead

$$a_0 + 14609\cdot a_1 \equiv 21,90\ \ (28,126) = \frac{38}{49}.$$

Remark 12. For comparisons with modern astronomical calculations, note that the anomaly is measured from the Moon's apogee, while Western astronomy measures it from the perigee, which makes the values differ by a half-circle.

The equation of the moon (*zla rkang*) is calculated by

$$moon\_equ := moon\_tab(28 \cdot anomaly\_moon)$$

where $moon\_tab(i)$ is listed in the following table for $i = 0, \dots, 7$, which extends by the symmetry rules $moon\_tab(14-i) = moon\_tab(i)$, $moon\_tab(14+i) = -moon\_tab(i)$, and thus $moon\_tab(28+i) = moon\_tab(i)$; linear interpolation is used beween integer arguments.

i	0	1	2	3	4	5	6	7
$moon\_tab(i)$	0	5	10	15	19	22	24	25

To find the equation of the sun (*nyi rkang*), first calculate the anomaly by

$$anomaly\_sun := mean\_sun - 1/4 \tag{49}$$

and then take

$$sun\_equ := sun\_tab(12 \cdot anomaly\_sun)$$

where $sun\_tab(i)$ is listed in the following table for $i = 0, \dots, 3$, which extends by the symmetry rules $sun\_tab(6-i) = sun\_tab(i)$, $sun\_tab(6+i) = -sun\_tab(i)$, and thus $sun\_tab(12+i) = sun\_tab(i)$; linear interpolation is used beween integer arguments.

i	0	1	2	3
$sun\_tab(i)$	0	6	10	11

The date at the end of the lunar day (*gza' dag*) is finally calculated as

$$true\_date := mean\_date + moon\_equ/60 - sun\_equ/60. \tag{50}$$

(The half-corrected $mean\_date + moon\_equ/60$ is called semi-true date (*gza' phyed dag pa*).)

Similarly, although not needed to calculate the calendar date, the true solar longitude (*nyi dag*) is

$$true\_sun := mean\_sun - sun\_equ/(27 \cdot 60). \tag{51}$$

Remark 13. You will not find the factors $1/60$ and $1/(27 \cdot 60)$ explicit in the references; they are consequences of the positional system with mixed radices. Furthermore, $1/4$ in (49) is traditionally expressed as $6, 45\ \ (27, 60)$.

Remark 14. We have $m_2 = m_1/30$ and $s_2 = s_1/30$, which is very natural, since it means that the functions $mean\_date$ and $mean\_sun$ are linear functions of the lunar day count $d + 30 \cdot n$, without any jumps at the beginning of a new month. For the anomaly, however, the standard value $a_2 = 1/28$ does not conform to this. Note that a_1 could be replaced by $1 + a_1 = 30, 1\ \ (28, 126)$ (since we count modulo 1 here), and then $a_2 = 1, 0\ \ (28, 126) = 1/28$ is a close approximation to $(1 + a_1)/30$; the conclusion is that one usually for convenience uses the rounded value $a_2 = 1/28$. Henning [3], however, uses instead the value

$$a_2 := \frac{1 + a_1}{30} = \frac{3781}{105840} = \frac{1}{28} + \frac{1}{105840} = 1, 0, 1\ \ (28, 126, 30);$$

this is also used in his calendars [12]. The difference $1/105840$ is small, and the resulting difference in the anomaly is at most $30/105840 = 1/3528$; the difference in the argument to *moon_tab* is thus at most $28/3528 = 1/126$; since the increments in *moon_tab* are at most 5, the difference in *moon_equ* is at most $5/126$; finally, by (50), the difference in the true date is at most $(5/126)/60 = 1/1512$, so when rounding to an integer (see (52) below), we would expect to obtain the same result except, on the average, at most once in 1512 days. We would thus expect that the two different values of a_2 would give calendars that differ for at most one day in 1512 on the average. Moreover, since this was a maximum value, and the average ought to be less by a factor of about $1/2$, or more precisely $15.5/30 = 31/60$, since the difference is proportional to the day d which on the average is 15.5, and by another factor of $5/7$ since the average derivative of *moon_tab* is $25/7$ while we just used the maximum derivative 5. (For a sine function, the average absolute value of the derivative is $2/\pi$ times the maximum value, but the ratio is closer to one for the approximation in *moon_tab*.) Hence we would expect the average (absolute) difference in *true_date* to be about

$$\frac{31}{60} \cdot \frac{5}{7} \cdot \frac{1}{1512} \approx \frac{1}{4100}.$$

Consequently, we expect that the two versions of a_2 will lead to different Tibetan dates for, on the average, one day in 4200, or roughly one day in 10 years. A computer search has found the recent examples 10 February 2001 (JD 2451951) and 10 May 2006 (JD 2453866); the next example is 19 November 2025 (JD 2460999). It would be interesting to check these dates in published calendars.

9. Calendrical functions

As explained in Section 7, the Julian day number JD of a given Tibetan date can be calculated as the JD of the calendar day containing the end of the corresponding lunar day, i.e.

$$\mathrm{JD} := \lfloor \mathit{true\_date} \rfloor, \tag{52}$$

with *true_date* given by (50), except that if the Tibetan date is repeated, this gives the JD of the later day; for the earlier we thus have to subtract 1. If we do the calculations for a day that is skipped, formula (52) will still give the JD of the calendar day that lunar day ends, which by the rule in Section 7 is the day with name of the preceding lunar day (since that lunar day ends the same calendar day).

The calendar is really given by the inverse of this mapping; a day is given the number of the corresponding lunar day; if no such lunar day exists, it gets the same number as the next day. To find the Tibetan date for a given JD, we thus compute approximate month count and day (using the mean motion in (38)) and then search the neighbouring lunar days for an exact match, if any, taking care of the special cases when there are 0 or

2 such lunar days. For a detailed implementation, see the third edition of Dershowitz and Reingold [1].

Beginning and end of months. To find the last day of a month, we can just compute the JD for lunar day 30 of the month; this gives the correct result also when day 30 is repeated or omitted. To find the first day, however, requires a little care since lunar day 1 may end during the second day of the month (when day 1 is repeated) or during the last day of the preceding month (when day 1 is omitted); the simplest way to find the JD of first day of a month is to add 1 to the JD of the last day of the preceding month. By the comment on skipped days above, this can be computed as 1 + the JD of day 30 the preceding month, regardless of whether day 30 is skipped or not. (There seems to be errors in the tables in [4] due to this.)

Tibetan New Year. The Tibetan New Year (*Losar*) is celebrated starting the first day of the year. Since the first month may be a leap month 1 (which happens twice every 65 year cycle, the last time in 2000) and the first day may be day 2 (when day 1 is skipped), some care has to be taken to calculate the date. The simplest is to add 1 to the JD of the last day the preceding year, which thus is 1 + JD of the last day of (regular) month 12 the preceding year (and can be calculated as 1 + JD of day 30 of (regular) month 12 the preceding year).

Remark 15. Holidays are otherwise usually not celebrated in leap months. However, the Tibetan New Year is really the first day of the year even in a year that begins with a leap month 1. This is verified by [10] (written by a representative of the personal monastery of the Dalai Lama so presumably correct), according to which *Losar* was celebrated on Sunday, February 6th (2000), which was the first day of leap month 1.

10. Day of week

Each calendar day is, as explained in Sections 7 and 9, given a number in the range 1,...,30. It also has a day of week, which as in the Gregorian and many other calendars simply repeats with a period of 7. The day of week thus corresponds uniquely to the Western day of week.

The days of week are numbered 0,...,6, and they also have names. The correspondence with the English names is given in Table 3. (For the last column, see Appendix D.)

The day of week is simply calculated from the Julian day number found in (52) by

$$day\_of\_week := \text{JD} \mod 7. \tag{53}$$

The date (i.e., the number of the day) and the day of week are often given together. This resolves the ambiguity when days are repeated. (It also helps to resolve most ambiguities when different rules of calculation may ahve been used.)

	English	Tibetan	element
0	Saturday	spen ma	earth
1	Sunday	nyi ma	fire
2	Monday	zla ba	water
3	Tuesday	mig dmar	fire
4	Wednesday	lhag pa	water
5	Thursday	phur bu	wind
6	Friday	pa sangs	earth

Table 3. Days of week.

11. Further calculations

Tibetan calendars traditionally also contain further information, for astrological purposes, see Henning [3, Chapter IV] and, for an extensive example, [12]. These include (sometimes, at least) the following. (The numbers are usually truncated to 3 significant terms.) See further Appendix D.

- The true day of week, i.e. the day of week and fractional part of the day when the lunar day ends. This is $(true\_date + 2) \mod 7$, written with the radices (60,60,6,707).
- The longitude of the moon at the end of the lunar day (*tshes khyud*), calculated for lunar day d, month count n by
$$moon\_lunar\_day(d, n) := true\_sun(d, n) + d/30 \tag{54}$$
and written in lunar mansions with the radices (27,60,60,6,67).
- The longitude of the moon at the beginning of the calendar day (*zla skar*) calculated from the values in (54) and (50) by
$$moon\_calendar\_day := moon\_lunar\_day - \operatorname{frac}(true\_date)/27, \tag{55}$$
and written in lunar mansions with the radices (27,60,60,6,67).
- The name of the lunar mansion (Sanskrit *naksatra*), which is determined by the same moon longitude with fractions of mansions ignored. More precisely, numbering the lunar mansions from 0 to 26, the number of the lunar mansion is
$$\lfloor 27 moon\_calendar\_day \rfloor. \tag{56}$$
The names of the mansions (in Tibetan and Sanskrit) are listed in [3, Appendix I].
- The longitude of the sun (*nyi dag*), given by *true_sun* and written in mansions with the radices (27,60,60,6,67). (Note that *true_sun* really is computed for the end of the lunar day; the motion of the sun during the day is thus ignored.)
- The mean longitude of the sun in signs and degrees (and minutes), i.e. *mean_sun* written with the radices (12,30,60).

- The yoga "longitude" (*sbyor ba*) is the sum of the longitudes of the sun and the moon, calculated by

$$yoga\_longitude := moon\_calendar\_day + true\_sun \pmod 1, \tag{57}$$

  and written in mansions with the radices (27,60,60,6,67).
- The name of the yoga, which is determined by the yoga longitude with fractions of mansions ignored. More precisely, numbering the yogas from 0 to 26, the number of the yoga is

$$\lfloor 27 yoga\_longitude \rfloor. \tag{58}$$

  The names of the yogas (in Tibetan and Sanskrit) are listed in [3, Appendix I].
- The *karana* in effect at the start of the calendar day. Each lunar day is divided into two halves, and each half-day is assigned one of 11 different karanas. There are 4 "fixed" karanas that occur once each every month: the first half of the first lunar day, the second half of the 29th lunar day, and the two halves of the 30th day; the remaining 7 karanas, called "changing", repeat cyclically for the remaining 56 half-days. In other words, lunar day D consists of half-days $2D-1$ and $2D$, and half-day H has one of the fixed karanas if $H = 1, 58, 59, 60$, and otherwise it has the changing karana number $(H-1)$ amod 7. The names of the karanas (in Tibetan and Sanskrit) are listed in [3, Appendix I]. (The exact rule for determining the time in the middle of the lunar day that divides it into two halves is not completely clear to me. Henning [3] divides each lunar day into two halves of equal lengths, but there may be other versions.)

12. Holidays

A list of holidays, each occuring on a fixed Tibetan date every year, is given in [3, Appendix II]. If a holiday is fixed to a given day, and that day is skipped, the holiday is on the preceding day. If the day appears twice, the holiday is on the first of these (i.e., the second is regarded as the leap day). (These rules are given by [7], but I have not checked them against printed calendars.)

13. Mean lengths

The mean length of the month is

$$m_1 = 29; 31, 50, 0, 480 \ \ (60, 60, 6, 707) = \frac{167025}{5656} \approx 29.530587 \text{ days}, \tag{59}$$

which is essentially identical to the modern astronomical value of the synodic month 29.5305889 (increasing by about $2 \cdot 10^{-7}$ each century) [2, (12.11-2)].

Consequently, the mean length of the lunar day is

$$\frac{m_1}{30} = \frac{167025}{30 \cdot 5656} = \frac{11135}{11312} \approx 0.98435 \text{ days}. \tag{60}$$

The mean length of the year is, cf. (18),

$$\frac{1}{s_1}\text{ months} = \frac{m_1}{s_1} = \frac{804}{65}m_1 = \frac{6714405}{18382} \approx 365.270645 \text{ days}, \tag{61}$$

which is 0.02846 days more than the modern astronomical value of the tropical year 365.24219 [2, (12.11-1) and Table 15.3], and 0.02815 days longer than the mean Gregorian year 365.2425 days. (It is also longer than the sidereal year 365.25636 days [2, Table 15.3].) Hence the Tibetan year lags behind and starts on the average later, compared to the seasons or the Gregorian year, by almost 3 days per century or almost a month (more precisely, 28 days) per millennium.

The year has thus drifted considerably since it was introduced, and even more since the Kalacakra Tantra was written; the drift during the 1200 years since the epoch 806 is 34 days. Indeed, the solar longitude as computed by *true_sun* above will pass 0 during the 9th day in the third Tibetan month 2007, which equals April 25, 35 days after the astronomical vernal equinox on March 21, which agrees well with this drift.

The mean length of the anomalistic month is

$$\frac{1}{1+a_1}\text{ months} = \frac{3528}{3781}\cdot\frac{167025}{5656} = \frac{10522575}{381881} \approx 27.55459 \text{ days}. \tag{62}$$

14. Period

The *mean_date* given by (38) repeats after 5656 months, but *true_date* depends also on *anomaly_moon* and *true_sun*, which repeat after 3528 and 804 months, respectively. The least common multiple of 5656, 3528 and 804 is $p := 23873976$; consequently, all astronomical functions above repeat after p months, which equals $m_1 p = 705012525$ days or $s_1 p = 1930110$ years (these are necessarily integers). Note further that this period contains an integral number of leap year cycles (804 months = 65 years), see Section 6, so the numbering of the months repeats too. In other words:

The calendar (days and months) repeats after

$$705\,012\,525 \text{ } \textit{days} = 23\,873\,976 \text{ } \textit{months} = 1\,930\,110 \text{ } \textit{years}.$$

Moreover, the number of days is divisible by 7, so also the day of week repeats after this period.

However, $1930110 \equiv 6 \pmod{12}$, so to repeat the animal names of the years in the 12-year cycle, or the names in the 60-year cycle, we need two of these periods, i.e. 3 860 220 years.

Of course, the period is so long that the calendar will be completely out of phase with the tropical year and thus the seasons long before, and it will move through the seasons hundreds of times during one period.

If we also consider the planets, see Appendix C, the period becomes a whooping 2 796 235 115 048 502 090 600 years, see Henning [3, pp. 332–333].

Appendix A. The 60 year cycle

The 60 year cycle is derived from an Indian cycle of 60 names for the "Jovian years". (It takes Jupiter almost 12 years to orbit the sun. A 1/12 of the orbital period can be called a Jovian year, and the traditional Indian Jovian cycle gives each Jovian year a name (*samvatsara*) from a list of 60 names, so the names repeat with a cycle of 5 revolutions. The solar years are given the same names, based on the calculated position of Jupiter at the beginning of the year; hence sometimes (every 85 or 86 years) a name is omitted "expunged" from the list. In southern India this has from the 9th century been simplified to a simple 60 year cycle of names, and the same is done in Tibet. [1], [3, p. 143f].) Table 4 (taken from [3]) gives the full list of names, in Tibetan and Sanskrit, together with the corresponding Gregorian years in the last and current cycle. (Somewhat different transliterations of the Sanskrit names are given in [1].)

Table 5 gives the Gregorian dates of the new year for the years in the last and current cycles.

year		element–animal	Tibetan	Sanskrit		
1	4	Fire–Rabbit	rab byung	prabhava	1927	1987
2	5	Earth–Dragon	rnam byung	vibhava	1928	1988
3	6	Earth–Snake	dkar po	suklata	1929	1989
4	7	Iron–Horse	rab myos	pramadi	1930	1990
5	8	Iron–Sheep	skyes bdag	prajapati	1931	1991
6	9	Water–Monkey	anggi ra	ankira	1932	1992
7	10	Water–Bird	dpal gdong	srimukha	1933	1993
8	11	Wood–Dog	dngos po	bhava	1934	1994
9	12	Wood–Pig	na tshod ldan	yuvika	1935	1995
10	13	Fire–Mouse	'dzin byed	dhritu	1936	1996
11	14	Fire–Ox	dbang phyug	isvara	1937	1997
12	15	Earth–Tiger	'bru mang po	vahudhvanya	1938	1998
13	16	Earth–Rabbit	myos ldan	pramadi	1939	1999
14	17	Iron–Dragon	rnam gnon	vikrama	1940	2000
15	18	Iron–Snake	khyu mchog	brisabha	1941	2001
16	19	Water–Horse	sna tshogs	citra	1942	2002
17	20	Water–Sheep	nyi ma	bhanu	1943	2003
18	21	Wood–Monkey	nyi sgrol byed	bhanutara	1944	2004
19	22	Wood–Bird	sa skyong	virthapa	1945	2005
20	23	Fire–Dog	mi zad	aksaya	1946	2006
21	24	Fire–Pig	thams cad 'dul	sarvajit	1947	2007
22	25	Earth–Mouse	kun 'dzin	sarvadhari	1948	2008
23	26	Earth–Ox	'gal ba	virodhi	1949	2009
24	27	Iron–Tiger	rnam 'gyur	vikrita	1950	2010
25	28	Iron–Rabbit	bong bu	khara	1951	2011
26	29	Water–Dragon	dga' ba	nanda	1952	2012
27	30	Water–Snake	rnam rgyal	vijaya	1953	2013

28	31	Wood–Horse	rgyal ba	jaya	1954	2014
29	32	Wood–Sheep	myos byed	mada	1955	2015
30	33	Fire–Monkey	gdong ngan	durmukha	1956	2016
31	34	Fire–Bird	gser ’phyang	hemalambha	1957	2017
32	35	Earth–Dog	rnam ’phyang	vilambhi	1958	2018
33	36	Earth–Pig	sgyur byed	vikari	1959	2019
34	37	Iron–Mouse	kun ldan	sarvavati	1960	2020
35	38	Iron–Ox	’phar ba	slava	1961	2021
36	39	Water–Tiger	dge byed	subhakrita	1962	2022
37	40	Water–Rabbit	mdzes byed	sobhana	1963	2023
38	41	Wood–Dragon	khro mo	krodhi	1964	2024
39	42	Wood–Snake	sna tshogs dbyig	visvabandhu	1965	2025
40	43	Fire–Horse	zil gnon	parabhava	1966	2026
41	44	Fire–Sheep	spre’u	pravamga	1967	2027
42	45	Earth–Monkey	phur bu	kilaka	1968	2028
43	46	Earth–Bird	zhi ba	saumya	1969	2029
44	47	Iron–Dog	thun mong	sadharana	1970	2030
45	48	Iron–Pig	’gal byed	virobhakrita	1971	2031
46	49	Water–Mouse	yongs ’dzin	paradhari	1972	2032
47	50	Water–Ox	bag med	pramadi	1973	2033
48	51	Wood–Tiger	kun dga’	ananda	1974	2034
49	52	Wood–Rabbit	srin bu	raksasa	1975	2035
50	53	Fire–Dragon	me	anala	1976	2036
51	54	Fire–Snake	dmar ser can	vingala	1977	2037
52	55	Earth–Horse	dus kyi pho nya	kaladuti	1978	2038
53	56	Earth–Sheep	don grub	siddhartha	1979	2039
54	57	Iron–Monkey	drag po	rudra	1980	2040
55	58	Iron–Bird	blo ngan	durmati	1981	2041
56	59	Water–Dog	rnga chen	dundubhi	1982	2042
57	60	Water–Pig	khrag skyug	rudhirura	1983	2043
58	1	Wood–Mouse	mig dmar	raktaksi	1984	2044
59	2	Wood–Ox	khro bo	krodhana	1985	2045
60	3	Fire–Tiger	zad pa	ksayaka	1986	2046

Table 4: The Chinese and Indian 60 year cycles of names, with the names from the Indian cycle both in Tibetan and in Sanskrit. The first number on each line shows the number in the Prabhava cycle; the second shows the number in the Chinese cycle. The last two numbers show the Gregorian years in the last and current cycles.

Appendix B. Different versions

Different traditions follow different rules for the details of the calculation of the Tibetan calendar, and as said in Section 1, there are at least two

4/3 1927	28/2 1987	Fire–Rabbit	2/3 1957	27/2 2017	Fire–Bird
22/2 1928	18/2 1988	Earth–Dragon	19/2 1958	16/2 2018	Earth–Dog
10/2 1929	7/2 1989	Earth–Snake	8/2 1959	5/2 2019	Earth–Pig
1/3 1930	26/2 1990	Iron–Horse	27/2 1960	24/2 2020	Iron–Mouse
18/2 1931	15/2 1991	Iron–Sheep	16/2 1961	12/2 2021	Iron–Ox
7/2 1932	5/3 1992	Water–Monkey	5/2 1962	3/3 2022	Water–Tiger
25/2 1933	22/2 1993	Water–Bird	24/2 1963	21/2 2023	Water–Rabbit
14/2 1934	11/2 1994	Wood–Dog	14/2 1964	10/2 2024	Wood–Dragon
4/2 1935	2/3 1995	Wood–Pig	4/3 1965	28/2 2025	Wood–Snake
23/2 1936	19/2 1996	Fire–Mouse	21/2 1966	18/2 2026	Fire–Horse
12/2 1937	8/2 1997	Fire–Ox	10/2 1967	7/2 2027	Fire–Sheep
3/3 1938	27/2 1998	Earth–Tiger	29/2 1968	26/2 2028	Earth–Monkey
20/2 1939	17/2 1999	Earth–Rabbit	17/2 1969	14/2 2029	Earth–Bird
9/2 1940	6/2 2000	Iron–Dragon	7/2 1970	5/3 2030	Iron–Dog
26/2 1941	24/2 2001	Iron–Snake	26/2 1971	22/2 2031	Iron–Pig
16/2 1942	13/2 2002	Water–Horse	15/2 1972	12/2 2032	Water–Mouse
5/2 1943	3/3 2003	Water–Sheep	5/3 1973	2/3 2033	Water–Ox
24/2 1944	21/2 2004	Wood–Monkey	22/2 1974	19/2 2034	Wood–Tiger
13/2 1945	9/2 2005	Wood–Bird	11/2 1975	9/2 2035	Wood–Rabbit
4/3 1946	28/2 2006	Fire–Dog	1/3 1976	27/2 2036	Fire–Dragon
21/2 1947	18/2 2007	Fire–Pig	19/2 1977	15/2 2037	Fire–Snake
10/2 1948	7/2 2008	Earth–Mouse	8/2 1978	6/3 2038	Earth–Horse
28/2 1949	25/2 2009	Earth–Ox	27/2 1979	23/2 2039	Earth–Sheep
17/2 1950	14/2 2010	Iron–Tiger	17/2 1980	13/2 2040	Iron–Monkey
7/2 1951	5/3 2011	Iron–Rabbit	5/2 1981	3/3 2041	Iron–Bird
26/2 1952	22/2 2012	Water–Dragon	24/2 1982	21/2 2042	Water–Dog
14/2 1953	11/2 2013	Water–Snake	13/2 1983	10/2 2043	Water–Pig
4/2 1954	2/3 2014	Wood–Horse	3/3 1984	29/2 2044	Wood–Mouse
23/2 1955	19/2 2015	Wood–Sheep	20/2 1985	17/2 2045	Wood–Ox
12/2 1956	9/2 2016	Fire–Monkey	9/2 1986	6/2 2046	Fire–Tiger

Table 5. Tibetan New Years (Phugpa) and year names for the last and current 60 year cycles.

versions in use today. (Several other versions survive according to [3, p. 9], but no details are given.) Schuh [4] gives historical information, including many versions of the constants in Section 8 above used or proposed during the centuries, but says nothing about the different versions today. Henning [3] discusses the Phugpa and Tsurphu versions in detail, and also one recent attempt at reform. Berzin [7] is another source on different versions today that I have found, and gives the impression that several versions are in use. However, since Berzin [7] discusses the calendar and astrology together, it is possible that some of the versions below actually use the same calendar but differ in other, astrological, calculations or interpretations. See also the web pages of Nitartha [9].

The original Kalacakra Tantra calculations are explained by Schuh [4, Chapter 5] and Henning [3, Chapter V], but they are (as far as I know) not used today.

Phugpa. (Phugluk) This is the most widespread version, and is regarded as the official Tibetan calendar, at least by some (including some followers of the Tsurphu tradition [9]; I doubt that it is official inside Tibet). It was started in 1447 and is based on works by Phugpa and others in the 15th century [3, p. 8], The Phugpa version is used by the Gelug, Sakya, Nyingma and Shagpa Kagyu traditions of Tibetan buddhism, including the Dalai Lama, and is used in the calendars published in Dharamsala in India, where the Tibetan exile government resides. The Bon calendar is the same as the Phugpa [7]. The Phugpa calendar is described in detail in the present paper.

Tsurphu. (Tsurluk, Karma Kagyu) This version was started at about the same time by Jamyang Dondrub Wozer and derives from 14th century commentaries to Kalacakra Tantra by the Third Karmapa Rangjung Dorjey, of Tsurphu monastery; the main seat of the Karma Kagyu tradition of Tibetan buddhism [7], [3, p. 9]. This version is used by the Karma Kagyu tradition. The calendar published by Nitartha in USA [6; 9] gives the Tsurphu version; since 2004 the Phugpa version too is given.

The Tsurphu tradition uses the astronomical functions in Section 8 with the same values as given there for the Phugpa version of the constants s_1, m_1 and a_1 (and s_2, m_2 and a_2, see Remark 14) for mean motions, while the epoch values s_0, m_0 and a_0 are different.

Remark 16. Traditional Tsurphu calculations use, however, somewhat different sets of radices that the Phugpa versions, with one more term and thus higher numerical accuracy. The same constants are thus written differently:

$$s_1 := 2, 10, 58, 1, 3, 20 \ \ (27, 60, 60, 6, 13, 67) = \frac{65}{804}, \tag{63}$$

$$m_1 := 29; 31, 50, 0, 8, 584 \ \ (60, 60, 6, 13, 707) = \frac{167025}{5656}, \tag{64}$$

although the traditional value of m_1 is $1; 31, 50, 0, 8, 584 \ \ (60, 60, 6, 13, 707)$, calculating modulo 7, see Remark 8.

Two epochs given in classical text are [3, p. 340], with m_0 modified to yield JD, see Remark 8:

JD = 2353745 (Wednesday, 26 March 1732 (Greg.))

$$s_0 := -\big(1, 29, 17, 5, 6, 1 \ \ (27, 60, 60, 6, 13, 67)\big) = -\frac{5983}{108540},$$

$$m_0 := 4; 14, 6, 2, 2, 666 \ \ (60, 60, 6, 13, 707) + 2353741 = 2353745 + \frac{1795153}{7635600}$$

$$a_0 := 14, 99 \ \ (28, 126) = \frac{207}{392}.$$

and 1485 months later (equivalent and giving identical results)

$$\mathrm{JD} = 2397598 \quad \text{(Monday, 19 April 1852)}$$

$$s_0 := 0, 1, 22, 2, 4, 18 \ \ (27, 60, 60, 6, 13, 67) = \frac{23}{27135},$$

$$m_0 := 2; 9, 24, 2, 5, 417 \ \ (60, 60, 6, 13, 707) + 2397596 = 2397598 + \frac{1197103}{7635600}$$

$$a_0 := 0, 72 \ \ (28, 126) = \frac{1}{49}.$$

We denote these by E1732 and E1852.

The intercalation index is given by (35) with $\beta^* = 59$ for E1732 and $\beta^* = 14$ for E1852. The Tsurphu intercalation rule differs from the Phugpa rule in Section 6 and is :

> *There is a leap month M in year Y if and only if the intercalation index $ix \equiv 0$ or $1 \pmod{65}$.*

This implies that for a regular month, the month count n is obtained from the true month (34) by rounding down to the nearest integer; for a leap month we further subtract 1. (The Tsurphu rules are thus simpler and more natural than the Phugpa rules in Section 6; they also follow the original Kalacakra Tantra.)

Corresponding to this difference in the rules, (33) is replaced by

$$\beta^* := (6 - \beta) \bmod 65; \tag{65}$$

the values for β are 142 (E1732) and 187 (E1852). Calculating backwards, this is consistent with, cf. (19) for Phugpa,

$$p_1 = 23, 0 \ \ (27, 60) = \frac{23}{27}, \tag{66}$$

which gives

$$\alpha = 12(s_0 - p_0) = 2 + \frac{1052}{9045} \qquad \text{(E1732)}, \tag{67}$$

$$\alpha = 12(s_0 - p_0) = 2 + \frac{7127}{9045} \qquad \text{(E1852)}. \tag{68}$$

Remark 17. Any p_1 satisfying

$$\frac{92432}{27 \cdot 4020} < p_1 \leq \frac{92567}{27 \cdot 4020} \tag{69}$$

would give the same β and β^* and thus the same formulas for true month and intercalation index. I do not know whether any of these values of p_1 really is specified in the Tsurphu tradition, or if the formulas are used without this interpretation.

The published Tsurphu calendars [6] show occasional minor differences in skipped and repeated days from the clendars in [12], which are calculated by Henning by the formulas and values given above. This suggests that there are several versions of the Tsurphuversion.

Drikung Kagyu. According to [7], the Drikung Kagyu tradition follows a system that combines the Tsurphu and Phugpa traditions.

Sherab Ling. A modern attempt at a reformed Tibetan calendar developed by Kojo Tsewang Mangyal (Tsenam) at the Sherab Ling monastery in Bir, India, is described by Henning [3, pp. 342–345]. It uses the value of m_1 in Section 8, but uses

$$s_1 = 2, 10, 58, 2, 564, 5546 \ \ (27, 60, 60, 6, 707, 6811) = \frac{3373793}{41232240}. \tag{70}$$

This deviation from the traditional value, and from the equivalent fundamental relation (18), means that leap months no longer will be regularly spaced in the traditional pattern with 2 leap months every 65 regular months.

Sawath. A version of the Sherab Ling calendar is published by the Jyotish Department of the Central Institute for Higher Tibetan Studies in Sarnath, India Henning [3, p. 346]. This version differs from all other Tibetan calendars that I have heard of in that the months begin at full moon (as in Indian calendars in northern India [1]).

Bhutan. Bhutan uses the Tibetan calendar; I think as the official calendar but I am not sure. According to [7], the Bhutanese version follows a derivative of the Tsurphu system called Cahtuhpitha–Kalachakra, started by Drugchen Pemakarpo in the late 16th century. It is used by the Drugpa Kagyu tradition. According to [7], the "main difference" from the Tsurphu system is that it is one day after it; if a day is the 9th in Tsurphu system, it is the 10th in the Bhutanese.

Sherpa. The Sherpas (a minority in Nepal) have a Tibetan calendar. The calendar for 2002 I found on the web [11] is identical to the Phugpa version, and this is possibly the general rule.

Mongolia. The Mongolian calendar is a version af the Tibetan. It was replaced by the Gregorian as the official calendar under communist rule in Mongolia; I do not know whether it has a revival now. At least the New Year is celebrated as a national holiday. According to Berzin [7], the Kalmyk Mongols in Russia follow the Phugpa version. The Khalka Mongols of Mongolia and the Buryats and Tuvinians of Siberia follow a variant of the Phugpa tradition known as New Geden or New Positive, which was started in 1786 by Sumpa Kenpo Yesheypejlor. Most of the calculations follow the same rules as the Phugpa system, but the starting point is the 40th year of a 60 year cycle. "Because of this difference, the Mongolian calendar works out to be unique." [7]

Yellow calculations. (Chinese-style) According to Berzin [7], the yellow system is like the Chinese in that it has no double or omitted days. Months have 29 or 30 days, numbered consecutively "and determined according to several traditions of calculation". The way it adds leap months is similar to, but not equivalent to, the Chinese. Unlike the Chinese calendar, it uses the basic calculations from Kalacakra Tantra. This seems to be a version of the Chinese calendar rather than the Tibetan. Inner Mongolia follows the yellow system [7]. See also [13].

Appendix C. Planets

The positions of the planets are, for astrological purposes, calculated by the following procedure, also based on the Kalacakra Tantra. The calculations yield the longitudes at the end of the calendar (=solar) day.

In modern terms, the (geocentric) longitude of a planet is found by first calculating the heliocentric longitude and the longitude of the sun, and then combining them (using trigonometry). The Tibetan calculations do effectively this, although the theory behind (which is not explicitly mentioned) rather is an old Indian version of the Ptolemaic system [3, p. 57]. As in the calculations of *true_date* in Section 8, the calculations use special tables as substitutes for trigonometrical calculations. For a more detailed description of the traditional calculation, including conversions between different radices, se Henning [3, Chapter II], which also includes discussions of the astronomical background and geometry, and of the accuracy of the formulas.

The constants given below are (as the main text of the present paper) for the Phugpa tradition; the Tsurphu version uses slightly different epoch values, see [3, p. 340].

General day. The general day (*spyi zhag*) is a count of (solar) days since the epoch. It is thus simply given by

$$general\_day := \mathrm{JD} - epoch. \tag{71}$$

For E1927 (as in [3]), the epoch is JD 2424972 (Friday, 1 April 1927; RD 703547) and thus

$$general\_day := \mathrm{JD} - 2424972. \tag{72}$$

Remark 18. The traditional method to calculate the general day is to first find the month count (= integer version of true month) n in Sections 4–6, see (15), (29), (37), and then use it to find the number of elapsed lunar days, ld say, since the epoch by $ld := 30n + D$, where D is the date.

Next, ld is multiplied by the ratio 11135/11312 in (60) (the mean length of a lunar day). A small constant gd_0 is subtracted; for E1927, $gd_0 = 2,178\ (64,707) = 199/5656$. This constant is the fraction of the solar day remaining at the end of the mean lunar day at the epoch. The difference $\frac{11135}{11312}ld - gd_0$ is rounded up to the nearest integer, and one sets provisionally

$$general\_day := \left\lceil \frac{11135}{11312}ld - gd_0 \right\rceil. \tag{73}$$

This gives an approximation of the general day, but may be off by a day because only the mean motion of the moon is considered (this is equivalent to ignoring the corrections in (50)). Hence the day of week is calculated by the simple formula

$$\big(general\_day + wd_0\big) \mod 7 \tag{74}$$

where the constant wd_0 is the day of week at the epoch; for E1927, $wd_0 = 6$. If the value in (74) differs from the correct day of week, then $general\_day$ is adjusted by ±1 (the error cannot be larger) so that (74) becomes correct. (Since this final check and correction has to be done, one could as well ignore the subtraction of gd_0 above, but it is traditionally done.)

Mean heliocentric motion. The mean heliocentric position of a planet is represented by an integer for each plane called *particular day* (*sgos zhay*), calculated as

$$particular\_day := \begin{cases} (100 \cdot general\_day + pd_0) \bmod R, & \text{Mercury}, \\ (10 \cdot general\_day + pd_0) \bmod R, & \text{Venus}, \\ (general\_day + pd_0) \bmod R, & \text{Mars, Jupiter, Saturn}, \end{cases} \tag{75}$$

where the modulus R and the epoch value pd_0 are given in Table 6. The periods of the planets are thus, exactly, 87.97, 224.7, 687, 4332 and 10766 days, respectively. (Modern astronomical values are 87.9684, 224.695, 686.93, 4330.6, and 10746.9 [2, Table 15.6].)

The particular day is 0 at the first point of Aries (i.e., when the longitude is 0), and thus the mean heliocentric longitude is

$$mean\_helio\_long := \frac{particular\_day}{R} \pmod 1. \tag{76}$$

This is traditionally expressed in the radices $(27, 60, 60, 6, r)$ with the final radix r depending on the planet and given in Table 6. (Note that r always is a divisor of R.)

	Mercury	Venus	Mars	Jupiter	Saturn
R	8797	2247	687	4332	10766
pd_0 (E1927)	4639	301	157	3964	6286
r	8797	749	229	361	5383
$birth\_sign$	11/18	2/9	19/54	4/9	2/3
trad. (27,60)	16,30	6,0	9,30	12,0	18,0

TABLE 6. Constants for planets.

Mean longitude of the sun. The mean longitude of the sun (at the end of the calendar day) is calculated from scratch and not using the calculation of $mean\_sun$ (at the end of the lunar day) in Section 8, but the results are consistent [3, pp. 87–88]. (The Tsurphu tradition uses the calculation of *mean_sun* [3, p. 341]; I do not know whether it is corrected for the time difference between the end of lunar and solar days, but I guess that it is not, and used as is.) The formula used is

$$mean\_solar\_long := s_1' \cdot general\_day + s_0', \tag{77}$$

where

$$s_1' = 0,4,26,0,93156 \ \ (27,60,60,6,149209) = \frac{18382}{6714405} \quad \Big(= \frac{11312}{11135} s_2\Big) \tag{78}$$

and the epoch value is, for E1927,

$$s_0' = 25,9,20,0,97440 \ \ (27,60,60,6,149209) = 1 - \frac{458772}{6714405}. \tag{79}$$

	Mercury	Venus	Mars	Jupiter	Saturn
0	0	0	0	0	0
1	10	5	25	11	22
2	17	9	43	20	37
3	20	10	50	23	43

TABLE 7. Equation for planets.

	Mercury	Venus	Mars	Jupiter	Saturn
0	0	0	0	0	0
1	16	25	24	10	6
2	32	50	47	20	11
3	47	75	70	29	16
4	61	99	93	37	20
5	74	123	114	43	24
6	85	145	135	49	26
7	92	167	153	51	28
8	97	185	168	52	28
9	97	200	179	49	26
10	93	208	182	43	22
11	82	202	171	34	17
12	62	172	133	23	11
13	34	83	53	7	3

TABLE 8. Final correction for planets.

Slow longitude and step index. The remaining calculations are based on the mean heliocentric longitude and the mean solar longitude, but these quantities are treated differently for the inner (or "peaceful") planets Mercury and Venus and for the outer (or "wrathful") planets Mars, Jupiter, Saturn. The reason is that the mean motion of an outer planet is given by the mean heliocentric longitude, while the mean motion of an inner planet is given by the mean longitude of the sun. In both cases, this main term is called the *mean slow longitude* (*dal ba*), while the other quantity is called the *step index* (*rkang 'dzin*). In other words, for the inner planets

$$\begin{aligned} mean\_slow\_long &:= mean\_solar\_long, \\ step\_index &:= mean\_helio\_long \end{aligned}$$

and for the outer planets

$$\begin{aligned} mean\_slow\_long &:= mean\_helio\_long, \\ step\_index &:= mean\_solar\_long. \end{aligned}$$

Next, cf. the calculations for the moon and sun in Section 8, the anomaly is calculated by

$$anomaly := mean\_slow\_long - birth\_sign \pmod 1, \tag{80}$$

where the "birth-sign" (*skyes khyim*) is given in Table 6, both as a rational number and in the traditional form in mansions (27,60). The anomaly is used to find the equation from

$$equ := planet\_equ\_tab(12 \cdot anomaly), \tag{81}$$

where $planet\_equ\_tab(i)$ is given in Table 7 for $i = 0, \ldots, 3$, which extends by the symmetry rules $planet\_equ\_tab(6 - i) = planet\_equ\_tab(i)$ and $planet\_equ\_tab(6 + i) = -planet\_equ\_tab(i)$; linear interpolation is used beween integer arguments. Finally, the true slow longitude (*dal dag*) is given by

$$true\_slow\_long := mean\_slow\_long - equ/(27 \cdot 60). \tag{82}$$

Geocentric longitude. The final step is to combine the true slow longitude and the step index. First, the difference of these is found:

$$diff := step\_index - true\_slow\_long. \tag{83}$$

This is used to find a correction by another table look-up:

$$corr := planet\_corr\_tab(27 \cdot diff) \pmod 1, \tag{84}$$

where $planet\_corr\_tab(i)$ is given in Table 8 for $i = 0, \ldots, 13$, which extends by the symmetry rules $planet\_corr\_tab(27 - i) = -planet\_corr\_tab(i)$ and $planet\_corr\_tab(27 + i) = planet\_corr\_tab(i)$; as always, linear interpolation is used beween integer arguments.

Finally the (geocentric, or *fast*) longitude (*myur ba*) is given by

$$fast\_long := true\_slow\_long + corr/(27 \cdot 60). \tag{85}$$

Rahu. *Rahu* is the name of the nodes of the lunar orbit, i.e., the intersections of the orbit and the ecliptic. More precisely, the ascending node is called the Head of Rahu and the descending node is called the Tail of Rahu. In Tibetan (as in Indian) astrology, Rahu is treated as a planet, or perhaps two planets. Rahu is further essential for prediction of eclipses [3, Chapter III].

Rahu has a slow motion that is retrograde (i.e., to the west, with decreasing longitude, unlike the real planets). In the Tibetan system, the period is exactly 230 lunar months = 6900 lunar days.

Remark 19. In calendar (solar) days, this is, cf. (60),

$$\frac{11135}{11312} \cdot 6900 = \frac{19207875}{2828} \approx 6792.035 \text{ days} \tag{86}$$

or, cf. (61),

$$\frac{11135}{11312} \cdot 6900 s_2 = 6900 \cdot \frac{s_1}{30} = \frac{7475}{402} \approx 18.5945 \text{ Tibetan years.} \tag{87}$$

The modern astronomical value is 6798 days = 18.61 Georgian years [2, Table 15.4].

To find the position of Rahu at day D, month M, year Y, one first calculates the month count (i.e., integer version of true month) n by (15) in Section 4. Next, the number x of elapsed lunar days since the Head of Rahu had longitude 0 is calculated by, cf.the calculation of ld in Remark 18,

$$x := 30(n + rd_0) + D = 30n + D + 30rd_0 = ld + 30rd_0, \tag{88}$$

where rd_0 is an epoch value; $rd_0 = 187$ for E1927. (For E1987, $rd_0 = 10$.) Finally, the longitudes of the head and tail of Rahu are given by

$$\mathit{rahu\_head\_long} := -\frac{x}{6900} \pmod 1, \tag{89}$$

$$\mathit{rahu\_tail\_long} := \mathit{rahu\_head\_long} + \frac{1}{2} \pmod 1. \tag{90}$$

Remark 20. Since Rahu has a retrograde motion, these are decreasing. In traditional calculations, $-\mathit{rahu\_head\_long} := x/6900 \pmod 1$ is called the longitude of the Source of Rahu. The longitudes are traditionally expressed in $(27, 60, 60, 6, 23)$, and $1/6900$ is then written as $0, 0, 14, 0, 12$.

Appendix D. Further astrological calculations

As explained in Section 3, each year has a name consisting of an element and an animal. For astrological purposes (in the Chinese or elemental astrological system), there are many further associations, assigning a year, month, lunar day or solar day one of the 12 animals in Table 2, one of the 5 elements in Table 9, one of the 8 trigrams in Table 10, or one of the 9 numbers $1, \dots, 9$ in Table 11; as shown in Tables 2, 9, 10, 11, these have further associations to, for example, numbers, colours and directions. There are also further attributes in the Indian system. (Only some of the attributes are mentioned here. See Henning [3] for further details.)

number	element	colour
1	wood	green (blue)
2	fire	red
3	earth	yellow
4	iron	white
5	water	blue (black)

Table 9. The 5 elements and the associated numbers and colours.

	binary	trigram	Tibetan	Chinese	direction	element
1	5	☲	li	lí	S	fire
2	0	☷	khon	kūn	SW	earth
3	6	☱	dwa	duì	W	iron
4	7	☰	khen	qián	NW	sky
5	2	☵	kham	kǎn	N	water
6	1	☶	gin	gèn	NE	mountain
7	4	☳	zin	zhèn	E	wood
8	3	☴	zon	xùn	SE	wind

Table 10. The 8 trigrams with some attributes. The ordering is the standard (King Wen, Later Heaven) order; the numbering is perhaps not traditional, and the binary coding (reading the trigrams bottom-up) is mathematically natural but not traditionally used.

	colour	element	direction
1	white	iron	N
2	black	water	SW
3	blue	water	E
4	green	wood	SE
5	yellow	earth	Centre
6	white	iron	NW
7	red	fire	W
8	white	iron	NE
9	red	fire	S

Table 11. The 9 numbers and their attributes.

Remark 21. The order of the directions in Table 11 may seem jumbled, yet there is method in it. The 9 numbers are often arranged in a 3×3 square according to the directions as in Table 12 (upside down the standard

Western orientation), and then the numbers form a magic square with all rows, columns and diagonals summing to 15.

4	9	2
3	5	7
8	1	6

SE	S	SW
E	C	W
NE	N	NW

TABLE 12. A magic square of numbers and their directions

In formulas below, Y is the Gregorian number of the year, see Section 3. By Section 3, year Y has in the Chinese 60 year cycle number

$$z := (Y - 3) \ \text{amod}\ 60, \tag{91}$$

and hence numbers $(Y-3) \ \text{amod}\ 10$ and $(Y-3) \ \text{amod}\ 12$ in the Chinese 10 and 12 year cycles.

Attributes for years.

Elements. Each year is given 4 or 5 elements: the power element (*dbang thang*), life element (*srog*), body element (*lus*), fortune element (*klung rta*), and sometimes also the spirit element (*bla*).

The *power element* is the element associated to the celestial stem, see Table 1; it is thus repeated in a cycle of 10 years, with each element repeated 2 consecutive years, in the standard order wood, fire, earth, iron, water (the order in Table 9). As said above, year Y is $(Y-3) \ \text{amod}\ 10$ in the Chinese 10 year cycle, and by Tables 1 and 9, its power element has number

$$\left\lceil \frac{z}{2} \right\rceil = \left\lceil \frac{Y-3}{2} \right\rceil. \tag{92}$$

The *life element* is repeated in a cycle of 12 years, and is thus determined by the animal name of the year. The list is given in Table 13. Note that each third year is earth (the years $\equiv 2 \bmod 3$); the remaining four elements come repeated 2 years each, in the same (cyclic) order wood, fire, iron, water as the power element.

The *fortune element* is repeated in a cycle of 4 years, in the order wood, water, iron, fire. (Earth is not used. Note that the order of the four used elements is different from the order used for the power and life elements.) Since 4 is a divisor of 12, the fortune element is determined by the animal name of the year. See Table 14.

The *body element* is calculated in two steps. (See Henning [3] for traditional ways of doing the calculations.) First, an element is determined by the animal name; I do not know any name for this intermediate element so let us call it x. The element x is repeated in a cycle of 6 years, with the 3 elements wood, water, iron repeated 2 years each, see Table 15. Then count the number of steps from x to the power element y, or equivalently, replacing the elements by the corresponding numbers in Table 9, calculate $y-x \bmod 5$.

Finally, this difference determines the body element by Table 16. (Note that the order in this table is not the standard one.)

Since both x and the power element y are repeated 2 consecutive years each, the same is true for the body element. If we consider only the even years, say, then x is by Table 15 given by 1, 0, -1, 1, 0, -1, ... (mod 5), while y by Table 1 is given by 1, 2, 3, 4, 5, 1, 2, 3, ... Hence, $y - x \mod 5$ repeats in a cycle of length 15: 0, 2, 4, 3, 0, 2, 1, 3, 0, 4, 1, 3, 2, 4, 1 (with differences $+2, +2, -1, +2, +2, -1, \dots$).

Consequently, the body element repeats in a cycle of 30 years, with each element repeated for 2 consecutive years. The full cycle is given in Table 17.

The *spirit element* is the element preceding the life element in the standard order, see Table 13. Thus it too is repeated in a cycle of 12 years, and is determined by the animal name of the year. Each third year is fire (the years $\equiv 2 \bmod 3$) and the remaining four elements come repeated 2 years each, in the standard (cyclic) order.

Since all elements for the year have periods dividing 60, they all repeat in the same order in each 60 year cycle as is shown in Table 18.

Numbers. Each year is also associated with a set of three numbers, in the range $1, \dots, 9$, the *central number*, the *life number* and the *power number*. (These numbers are used for persons born that year. The central number is also called the *body number* or *birth number*.) The numbers are associated with elements and directions according to Table 11. The numbers decrease by 1 (mod 9) for each new year, and thus repeat in a cycle of 9 years; they are given simply by, for Gregorian year Y:

$$\text{central number} := (2 - Y) \ \operatorname{amod}\ 9, \tag{93}$$

$$\text{life number} := (\text{central number} - 3) \ \operatorname{amod}\ 9 = (8 - Y) \ \operatorname{amod}\ 9, \tag{94}$$

$$\text{power number} := (\text{central number} + 3) \ \operatorname{amod}\ 9 = (5 - Y) \ \operatorname{amod}\ 9. \tag{95}$$

Since 9 does not divide 60, these numbers do not follow the 60 year cycle. The period for repeating all elements and numbers is 180 years, i.e., 3 cycles of 60 years.

year		animal	life element	spirit element
1	10	Mouse	water	iron
2	11	Ox	earth	fire
3	12	Tiger	wood	water
4	1	Rabbit	wood	water
5	2	Dragon	earth	fire
6	3	Snake	fire	wood
7	4	Horse	fire	wood
8	5	Sheep	earth	fire
9	6	Monkey	iron	earth
10	7	Bird	iron	earth
11	8	Dog	earth	fire
12	9	Pig	water	iron

TABLE 13. The 12 year cycle of life elements. The first number on each line shows the year mod 12 counted from the start of a Chinese cycle; the second shows the year mod 12 counted from the start of a Prabhava cycle.

year		animals	fortune element
1	2	Mouse, Dragon, Monkey	wood
2	3	Ox, Snake, Bird	water
3	4	Tiger, Horse, Dog	iron
4	1	Rabbit, Sheep, Pig	fire

TABLE 14. The 4 year cycle of fortune elements. The first number on each line shows the year mod 4 counted from the start of a Chinese cycle; the second shows the year mod 4 counted from the start of a Prabhava cycle.

year		animals	element	number
1	4	Mouse, Horse	wood	1
2	5	Ox, Sheep	wood	1
3	6	Tiger, Monkey	water	5
4	1	Rabbit, Bird	water	5
5	2	Dragon, Dog	iron	4
6	3	Snake, Pig	iron	4

TABLE 15. The 6 year cycle of the element x and its number in the body element calculation. The first number on each line shows the year mod 6 counted from the start of a Chinese cycle; the second shows the year mod 6 counted from the start of a Prabhava cycle.

$y - x \mod 5$	body element
0	iron
1	water
2	fire
3	earth
4	wood

TABLE 16. The final step in the body element calculation.

year		power	body element
1	28	wood	iron
2	29	wood	iron
3	30	fire	fire
4	1	fire	fire
5	2	earth	wood
6	3	earth	wood
7	4	iron	earth
8	5	iron	earth
9	6	water	iron
10	7	water	iron
11	8	wood	fire
12	9	wood	fire
13	10	fire	water
14	11	fire	water
15	12	earth	earth
16	13	earth	earth
17	14	iron	iron
18	15	iron	iron
19	16	water	wood
20	17	water	wood
21	18	wood	water
22	19	wood	water
23	20	fire	earth
24	21	fire	earth
25	22	earth	fire
26	23	earth	fire
27	24	iron	wood
28	25	iron	wood
29	26	water	water
30	27	water	water

TABLE 17. The 30 year cycle of the body element. The first number on each line shows the year mod 30 counted from the start of a Chinese cycle; the second shows the year mod 30 counted from the start of a Prabhava cycle.

year		name (power)	life	body	fortune	spirit
1	58	Wood–Mouse	water	iron	wood	iron
2	59	Wood–Ox	earth	iron	water	fire
3	60	Fire–Tiger	wood	fire	iron	water
4	1	Fire–Rabbit	wood	fire	fire	water
5	2	Earth–Dragon	earth	wood	wood	fire
6	3	Earth–Snake	fire	wood	water	wood
7	4	Iron–Horse	fire	earth	iron	wood
8	5	Iron–Sheep	earth	earth	fire	fire
9	6	Water–Monkey	iron	iron	wood	earth
10	7	Water–Bird	iron	iron	water	earth
11	8	Wood–Dog	earth	fire	iron	fire
12	9	Wood–Pig	water	fire	fire	iron
13	10	Fire–Mouse	water	water	wood	iron
14	11	Fire–Ox	earth	water	water	fire
15	12	Earth–Tiger	wood	earth	iron	water
16	13	Earth–Rabbit	wood	earth	fire	water
17	14	Iron–Dragon	earth	iron	wood	fire
18	15	Iron–Snake	fire	iron	water	wood
19	16	Water–Horse	fire	wood	iron	wood
20	17	Water–Sheep	earth	wood	fire	fire
21	18	Wood–Monkey	iron	water	wood	earth
22	19	Wood–Bird	iron	water	water	earth
23	20	Fire–Dog	earth	earth	iron	fire
24	21	Fire–Pig	water	earth	fire	iron
25	22	Earth–Mouse	water	fire	wood	iron
26	23	Earth–Ox	earth	fire	water	fire
27	24	Iron–Tiger	wood	wood	iron	water
28	25	Iron–Rabbit	wood	wood	fire	water
29	26	Water–Dragon	earth	water	wood	fire
30	27	Water–Snake	fire	water	water	wood
31	28	Wood–Horse	fire	iron	iron	wood
32	29	Wood–Sheep	earth	iron	fire	fire
33	30	Fire–Monkey	iron	fire	wood	earth
34	31	Fire–Bird	iron	fire	water	earth
35	32	Earth–Dog	earth	wood	iron	fire
36	33	Earth–Pig	water	wood	fire	iron
37	34	Iron–Mouse	water	earth	wood	iron
38	35	Iron–Ox	earth	earth	water	fire
39	36	Water–Tiger	wood	iron	iron	water
40	37	Water–Rabbit	wood	iron	fire	water
41	38	Wood–Dragon	earth	fire	wood	fire
42	39	Wood–Snake	fire	fire	water	wood
43	40	Fire–Horse	fire	water	iron	wood
44	41	Fire–Sheep	earth	water	fire	fire

45	42	Earth–Monkey	iron	earth	wood	earth
46	43	Earth–Bird	iron	earth	water	earth
47	44	Iron–Dog	earth	iron	iron	fire
48	45	Iron–Pig	water	iron	fire	iron
49	46	Water–Mouse	water	wood	wood	iron
50	47	Water–Ox	earth	wood	water	fire
51	48	Wood–Tiger	wood	water	iron	water
52	49	Wood–Rabbit	wood	water	fire	water
53	50	Fire–Dragon	earth	earth	wood	fire
54	51	Fire–Snake	fire	earth	water	wood
55	52	Earth–Horse	fire	fire	iron	wood
56	53	Earth–Sheep	earth	fire	fire	fire
57	54	Iron–Monkey	iron	wood	wood	earth
58	55	Iron–Bird	iron	wood	water	earth
59	56	Water–Dog	earth	water	iron	fire
60	57	Water–Pig	water	water	fire	iron

Table 18: The 60 year cycle of combinations of different elements. The first number on each line shows the year mod 60 counted from the start of a Chinese cycle; the second shows the year mod 60 counted from the start of a Prabhava cycle. The power element is the first part of the name.

Attributes for months.

Animals. The 12 months are assigned one each of the 12 animals, in standard order but with different starting points in the Phugpa and Tsurphu traditions. The full lists are given in Table 19. Leap months are (as far as I know) given the same animal as the regular month with the same number.

Quality. Each month is given the quality (male or female) associated to its animal. As Tables 2 and 19 show, this simply means (both in the Phugpa and Tsurphu traditions) that odd-numbered months are male and even-numbered female. (This is in accordance with the general Chinese principle that odd numbers are male (*yang*) and even numbers female (*yin*).)

Elements. The months are assigned elements, starting with the Tiger month, see Table 19; in the Tsurphu tradition this is month 1, but in the standard Phugpa version this is month 11 *the preceding year*.

In the Phugpa version, the Tiger month gets the element following the element of the year given in Table 1, using the standard element order in Table 9. (The element following another, x, in this cyclic order is called the *son* of x.) By (92), this is element $\lceil (Y-1)/2 \rceil \ \text{amod}\ 5$.

In the Tsurphu version, the Tiger month cycles through the 5 elements in order, and thus repeats in a cycle of 5 years. Year Y has Tiger month element $(z+1) \ \text{amod}\ 5 = (Y-2) \ \text{amod}\ 5$.

Having determined the element of the Tiger month, the elements for the 12 month period starting with it are assigned in the same pattern as for years in Table 1: each element is repeated 2 months, the first male and the second female, and then followed by the next element. (Again, I assume that leap months are ignored and receive the same element as the following regular month.) We thus obtain the following formulas for the number of the element for month M year Y.

Phugpa: If $M \geq 11$, then first replace $M := M - 12$ and $Y := Y + 1$.

$$\left(\left\lceil \frac{Y-1}{2} \right\rceil + \left\lfloor \frac{M+1}{2} \right\rfloor\right) \text{ amod } 5. \tag{96}$$

Tsurphu:

$$\left(Y - 2 + \left\lfloor \frac{M-1}{2} \right\rfloor\right) \text{ amod } 5. \tag{97}$$

month	Phugpa	Tsurphu	quality
1	Dragon	Tiger	male
2	Snake	Rabbit	female
3	Horse	Dragon	male
4	Sheep	Snake	female
5	Monkey	Horse	male
6	Bird	Sheep	female
7	Dog	Monkey	male
8	Pig	Bird	female
9	Mouse	Dog	male
10	Ox	Pig	female
11	Tiger	Mouse	male
12	Rabbit	Ox	female

Table 19. The animals for the months.

Numbers. Only Tsurphu calendars give one of the 9 numbers to each month. The number decreases by 1 (mod 9) for each month (except leap months), and thus by $12 \equiv 3 \pmod 9$ for each year. Month M year Y has number

$$\bigl(3 - (12Y + M)\bigr) \text{ amod } 9. \tag{98}$$

Attributes for lunar days.

Animal. Each lunar day has an animal. These repeat in the usual cycle of 12, see Table 2, with each odd-numbered month starting with Tiger (number 3 in Table 2) and each even-numbered month starting with Monkey (number 9 in Table 2); since there are exactly $30 \equiv 6 \pmod{12}$ lunar days in each month, the animals thus repeat in a continuous cycle broken only at leap months, where the animals are repeated in the same order in the leap month and the following regular month and there is a discontinuity between the two months. The number (in Table 2) of the animal for lunar day D in month M is thus

$$(D+30M+8) \ \text{amod}\ 12 = (D+6M+8) \ \text{amod}\ 12. \tag{99}$$

Element. Each lunar day has an element; these cycle inside each month in a cycle of 5 in the usual order given in Table 9 (without repetitions as for years and months), beginning with the element following the element of the month calculated above. If the month has element x, then lunar day D of the month thus has element $(x+D)$ amod 5. (There is thus often a jump in the sequence at the beginning of a new month.)

Trigram. The trigrams for lunar days cycle in the usual cycle of 8 in Table 10; as for the animals, this is continuous across months except for leap months. A Tiger month begins with trigram number 1, Li. Thus the trigram for lunar day D in a month with animal number A (in Table 2) has number

$$(D+30(A-3)) \ \text{amod}\ 8 = (D-2A-2) \ \text{amod}\ 8 = (D+6A+6) \ \text{amod}\ 8. \tag{100}$$

Hence months Tiger, Horse, Dog begin with Li; Rabbit, Sheep, Pig begin with Zin; Mouse, Dragon, Monkey begin with Kham; Ox, Snake, Bird begin with Dwa.

Number. The nine numbers (with their colours) for lunar days cycle forward in the usual cycle of 9 in Table 11; as for the animals, this is continuous across months except for leap months. A Tiger month begins with number 1 (white). Thus the number for lunar day D in a month with animal number A (in Table 2) has number

$$(D+30(A-3)) \ \text{amod}\ 9 = (D+3A) \ \text{amod}\ 9. \tag{101}$$

Hence, months Tiger, Snake, Monkey, Pig begin with 1 (white); Mouse, Rabbit, Horse, Bird begin with 4 (green); Ox, Dragon, Sheep, Dog begin with 7 (red).

Attributes for solar days. As explained in Sections 7, 9 and 10, each calendar (solar) day has a number (the date) and a day of week.

Each (solar) day is also given an element, animal, trigram and number from the Chinese system; these are simple cyclic with periods 10, 12, 8, 9, respectively, with the elements repeated twice each as for years in Table 1.

Element. The element corresponds to number JD amod 10 in the (Chinese) cycle in Table 1. The number of the element in Table 9 is thus

$$\left\lceil \frac{\text{JD} \ \text{amod}\ 10}{2} \right\rceil = \left\lceil \frac{\text{JD}}{2} \right\rceil \ \text{amod}\ 5. \tag{102}$$

Animal. The animal has number $(\text{JD}+2)$ amod 12 in the (Chinese) cycle in Table 2.

Trigram. The trigram has number $(\text{JD}+2)$ amod 8 in Table 10.

Number. The number in Table 11 is $(-\mathrm{JD}) \ \mathrm{amod}\ 9$.

Remark 22. Henning [3, pp. 208–209] describe these using different, and presumably traditional, numberings for the 10 day cycle of elements and the 12 day cycle of animals; his numbers (for the same element and animal as given above) are $(\mathrm{JD}-2) \ \mathrm{amod}\ 10$ and $(\mathrm{JD}-2) \ \mathrm{amod}\ 12$. He further calculates the number as $10-((\mathrm{JD}+1) \ \mathrm{amod}\ 9)$.

The element calculated in (102) is the power element of the day. Exactly as for years, see above, further elements (life, fortune, body, spirit) can be calculated from the animal–element pair; these elements thus follow a cycle of 60 days, which is equal to the cycle in Table 18. A day has the elements given in Table 18 on line $(\mathrm{JD}-10) \ \mathrm{amod}\ 60$.

Remark 23. Tsurphu calendars use a different method to assign the nine numbers; the first Wood–Mouse day after the (true astronomical) winter solstice is 5 (yellow); then the numbers increase by 1 (mod 9) each day, until the first Wood–Mouse day after the (true astronomical) summer solstice, which is 4 (green), and then the numbers decrease by 1 (mod 9) each day for half a year until the first Wood–Mouse day after the next winter solstice. This requires accurate astronomical calculations of the solstices, which is a central part of the Chinese calendar system, but foreign to the Tibetan calendar calculations.

Elemental yoga. In the Indian system, each day of week has an associated element from the set {earth, fire, water, wind}, see Table 3. Further, each lunar mansion is also associated to an element from the same set, see Henning [3, Appendix I] for a list. Each calendar day is thus given a combination of two elements, for the day of week and for the lunar mansion (calculated in (53) and (56)); the order of these two elements does not matter and the combination is regarded as an unordered pair. There are thus 10 possible different combinations (*yogas*), each having a name, see [3, p. 204].

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