

## Corrigendum to: The sum of powers of subtree sizes for conditioned Galton–Watson trees

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### Abstract

A serious typo in [1] is corrected.

**Keywords:** Conditioned Galton–Watson tree; simply generated random tree; additive functional; tree recurrence; subtree sizes; Brownian excursion; random analytic function; generating function; singularity analysis; Hadamard product of sequences; method of moments; polylogarithm.

**MSC2020 subject classifications:** Primary 05C05, Secondary 60F05; 60C05; 30E99.

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## 1 Corrigendum

There is a typo in [1, Theorem D.1]; the variance given in (D.2) there is incorrect and should be

$$\mathbb{E}|\zeta|^2 = \frac{1}{2\sqrt{\pi}} \operatorname{Re} \frac{\Gamma(it - \frac{1}{2})}{\Gamma(it)}. \quad (1.1)$$

The formula (D.2) in [1] has, incorrectly,  $\Gamma(it - 1)$  in the denominator, which comes from (D.5) which has the same error. Formula (D.8) in the proof is correct, with denominator  $\Gamma(it)$ , and yields (D.5) and (D.2) with the same denominator, i.e., (1.1).

Theorem D.1 in [1] also claims that  $\mathbb{E}|\zeta|^2 > 0$ . The proof is based on the incorrect formula given there, but luckily the same proof applies also to the correct formula. In (D.14) we obtain  $\Gamma(1 - it)$  instead of  $\Gamma(2 - it)$  (and an immaterial change of sign); hence we have to show that  $\Gamma(1 - it)/\Gamma(\frac{3}{2} - it)$  is not real for  $t \neq 0$ . Thus, in (D.15), we should have  $-\operatorname{Im} \int_1^{3/2} \psi(s - it) ds$ . We use (D.18) as before, and now see that if  $t < 0$ , then  $0 > \arg(\Gamma(1 - it)/\Gamma(\frac{3}{2} - it)) > -\pi/4$ , which completes the proof that the variance in (1.1) is nonzero.

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## Corrigendum

### 1.1 Other typos

We take the opportunity to point out a few other typos in [1]:

- (12.75): the lower summation limit should be  $m = 0$ .
- Three lines after (12.75): “every  $i$ ” should be “every “ $i \geq 1$ ”.
- Appendix C, two lines before the statement of Theorem C.1:  $\alpha^{-1}Y(\alpha)$  there should be  $\alpha^{-1/2}Y(\alpha)$ .

### References

- [1] James Allen Fill & Svante Janson. The sum of powers of subtree sizes for conditioned Galton-Watson trees. *Electron. J. Probab.* **27** (2022), Paper No. 114, 77 pp.