19 January 2009

What to study for the exam

In the exam, as you know, we do not ask hard mathematical questions. There are no proofs and no tricky questions. The exam is designed to be easy and mostly routine.

Many of you find it easy to learn a number of recipes and, often, apply them directly. You expect that the exam is similar to the one of the year before and the one of the year before that, and so on. To reply to your questions, indeed, this is the case.

So, typically, an exam will ask you to:

- perform some simple calculations
- find the stationary distribution of a simple chain
- examine the properties (such as irreducibility, aperiodicity) of a simple chain
- find the expected time it takes for a simple chain to reach a certain state
- work out the probability distribution of a simple branching process
- do some computations for a simple random walk.

As I said many times in class, the notes I have written constitute, more or less, a complete elementary course on Markov chains as it can be taught in a mathematical sciences department. I did include, both in the notes, and in my lectures, detailed explanations (proofs) of everything I taught. The reasons are these:

> (*) *It is impossible to learn anything unless you understand it; and by understanding I mean not to accept anything for granted but be in position to explain it.*
> (**) *I don't know how to teach in a so-called vocational manner. Moreover, I maintain that vocational teaching should not take place in universities.*

For instance, when I ask you, in a homework or in an exam, to find the stationary distribution of a Markov chain then most of you will know that you have to write and solve the equations

$$\pi(i) = \sum_j \pi(j) p_{ji},$$

and when I ask whether

$$P(X_n = i) \text{ converges to } \pi(i) \text{ as } n \to \infty,$$

most of you will know that you need to check irreducibility and aperiodicity. However, you should, ideally, be able to tell why. And, ideally, I should be allowed to ask you questions to test this understanding. Unfortunately, I will not. All you need to know is the result.

But I do hope that some of you will, as a result of my constant reminding of what constitutes learning, sit down and read the explanations. And I do appreciate that some of you do so! And do so not because you will earn any higher grade, but because you have convinced yourselves that (*) above holds. This is a tough call, because it is difficult to motivate yourselves to do something without reward. However, consider it as follows: the reward is not immediate, but it is a long-run award.

Here is a detailed list of the sections you need to study for the exam:

- Introduction §1
- Examples of Markov chains §2
- The Markov property §3
- Stationarity §4; omit example on bread kneading; pay attention to the writing the flow balance equations §4.1;
- Topological structure of Markov chain §5; learn the definitions and concepts
- First-step analysis §6; very important to understand how to write the equations
- Gambler's ruin §7
- Stopping times and the Markov property §8; unfortunately, we will not ask you proofs
- Recurrence and transience §9; regenerative structure §10; know the concepts well
- Positive recurrence §11; very important
- Law of large numbers §12; this was covered in separate lecture
- Law of large numbers for Markov chains §13; know how to apply it; unfortunately, we will not ask you proofs
- Construction of a stationary distribution §14; study it if you want to understand what is going on; in the exam, we cannot ask proofs
- Positive recurrence is a class property §15; I guess all you need to know is this statement by heart; understanding it is desirable, but we will not ask you proofs
- Uniqueness of stationary distribution §16; Know under what conditions
- Structure of a stationary distribution §17;
- Coupling and stability §18; only know the statements of results;
- Omit §19
- Ergodic theorems §20; know the results
- Finite chains §21; very small section
- Time reversibility §22; know the criteria
- New Markov chains from old ones §23; study this section
- Applications §24; learn the branching process example; read the others only if you need to practice–as additional exercises
- Random walk §25; know what it means
- The simple symmetric random walk in dimension 1: path counting §26; know it well
- The simple symmetric random walk in dimension 1: simple probability calculations §27; know it well

- The reflection principle §28; know what it is and how to apply it
- Urns and the ballot theorem §29; know what the ballot theorem says
- The asymmetric random walk in dimension 1 §30; know the concept only
- Recurrence properties of srw §31; know when a rw is recurrent/transient
- Omit §32, 33, 34, 35
- Appendices: Up to you to study to refresh background knowledge

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As for practice for the exam: Do all the homework sets again. Most of you never tried to do them, but if you decide to study them now, you should try to do them by yourselves. All the solutions are on the web page.

Past exams are posted on the web page.

Finally, a request: Please try to write neatly and logically. I do not appreciate pages and pages of calculations/explanations. Rather, concise and logical writing (in good English, not in mobile phone kind of English) is what I expect. As you can see from the solutions of past exams, the problems are rarely lengthy. The answers are short and rather simple.