HOMEWORK 1

1. _____

You toss a coin, independently from toss to toss, whose probability of heads is p and of tails $1 - p$. Find the expected number of tosses required to get the first head.

2. _____

Consider a box containing n balls, out of which m are red and $n - m$ blue. Start picking the balls one by one until you see a red one. What is the average number of picks required?

Assume, in particular, that n is large and that $m/n \to p$ as $n \to \infty$. Compare your result with the one of the previous problem.

3. _____

Explain what division of an integer n by a positive integer m means.

Divide the number 56793 by 382.

4. _____

Find the greatest common divisior between 56793 and 382.

5. _____

Show that the greatest common divisor between two positive integers m and n is an integer d such that (i) d divides both m and n and (ii) if k divides m and n then k divides d.

6. _____

Find the product AB of the matrices $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -3 \end{pmatrix}$.

Recall that, in general, $(AB)_{ij} = \sum_k A_{ik} B_{kj}$.

Can you compute BA also?