Department of Mathematics **Uppsala University**
Stochastic processes **Spring 2014**

Assignment 2
11 Feb. 2014

1. Fix $0 \le t \le \pi$ and define $f(s) := \min(s,t)$, $0 \le s \le \pi$. Show that the sequence of functions
$$f_n(s) := \frac{st}{\pi} + \frac{2}{\pi}\sum_{k=1}^{n} \frac{\sin(ks)\,\sin(kt)}{k^2}, \quad 0 \le s \le \pi,$$
converges, as $n \to \infty$, to the function f explaining in what sense(s) the convergence holds.

2. Explain the proof of Theorem 6.2. That is, understand it, and rewrite it in your own words.

3. Show that the law of iterated logarithm implies that, for all $t \ge 0$,
$$\mathbb{P}(W \text{ is not differentiable at } t) = 1.$$
In view of this, explain why Theorem 7.3, is strictly stronger.

4. Exercise 7.7.

5. Let X_t, $t \ge 0$, be a standard Poisson process (see Def. 5.1.) and let $T_1 < T_2 < \cdots$ be the points at which the process jumps. Let $Z_1, Z_2, \ldots$, be a sequence of i.i.d. standard normal random variables. Define
$$Y_t := \begin{cases} Z_1, & 0 \le t < T_1, \\ Z_k, & T_k \le t < T_{k+1}, \quad k = 1, 2, \ldots. \end{cases}$$
Show that, for all $t \ge 0$, Y_t is a Gaussian random variable, but that $(Y_t, t \ge 0)$ is not a Gaussian process. (See bottom of pg. 7 for the latter definition if you missed my lectures. Also, you need to know several things about Poisson process; or, read Ch. 5.)