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Title: Recent advances in ambit stochastics with a view towards tempo-spatial stochastic volatility/intermittency.

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Primary classification:

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Review text:

This paper offers a review of a certain class of random fields known as ambit fields that include processes of the form

$$Y_t(x) = \int_{A_t(x)} h(x, t; \xi, s)\, L(d\xi, ds)$$

where t is a temporal variable and x a spatial one, taking values in some set $\mathcal{X}$, usually, $\mathbb{R}^d$. The set $A_t(x) \subset \mathcal{X} \times \mathbb{R}$ is referred to as "ambit set". Randomness is mostly in the integrator $L(d\xi, ds)$ that is assumed to be a Lévy basis on $\mathcal{X} \times \mathbb{R}$. A Lévy basis L on a Borel space S equipped with a σ-finite reference measure λ is an independently scattered, infinitely divisible, random (signed) measure. In other words, (i) L is almost surely countably additive on the class of λ-finite measurable subset of S, (ii) L assigns independent random variables to disjoint λ-finite sets, and (iii) finite-dimensional distributions are infinitely divisible. Now, (ii) and (iii) are often intimately related, and that depends on the geometry of S, but, in general, one must assume them separately.

The paper gives an overview of the Lévy-Khinchine-type representation of a Lévy basis, followed by integration with respect to it. Specializing to ambit fields, the authors develop the concept of tempo-spatial stochastic volatility/intermittency and develop volatility modulation such as stochastic scaling of the amplitude and stochastic time change. Various properties are also discussed: smoothness of realizations and conditions that ensure that $t \mapsto Y_t(x)$ is a semimartingale. Special cases are studied. Particular attention is paid to so-called trawl processes defined by considering a fixed subset A of $\mathbb{R}^d \times \mathbb{R}$ with finite Lebesgue measure, a Lévy basis L on $\mathbb{R}^d \times \mathbb{R}$, and by letting

$$Y_t := L\{(x, t+s) : (x, s) \in A\}.$$

Various properties of trawl processes, such as their Lévy-Itô decomposition are discussed.

The reader should pay attention to the terminology employed in this paper. The term "random measure" allows for negative values, but the term "measure", when used in a deterministic context, is always a nonnegative measure. (The term "random signed measure" is reserved for objects that have a.s. finite variation.)