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This paper is concerned with arrays of $\{0,1\}$-valued random variables $[X_{i,j}^{(n)}]_{1\le i\le m_n, 1\le j\le n}$ such that, conditionally on i.i.d. random variables $\theta_1, \ldots, \theta_n$, the rows $X_{1,\bullet}^{(n)}, \ldots, X_{m_n,\bullet}^{(n)}$ are independent n-vectors, with the i-th row $X_{i,\bullet}^{(n)}$ being a sequence of Bernoulli(θ_i) random variables. The common distribution of the θ_i is a probability measure π_n on the interval $[0,1]$.

The authors prove that if $n\theta_n$ converges weakly to a random variable T, as $n \to \infty$, then $S_{n,i} := \sum_j X_{i,j}^{(n)}$ converges weakly to a random variable which is Poisson with mean T, conditionally on T. Also, if $nm_n\mathbb{E}\theta_n \to \lambda$ then $Z_{m_n,j} := \sum_i X_{i,j}^{(n)}$ converges to a Poisson(λ) limit. They then evaluate limit the limit of $S_{n,i}$ in several special cases. They also compute the limit of $\mathbb{P}(\max(S_1, \ldots, S_n) \le xb_n)$ for appropriate sequence b_n, and under certain conditions.

When $m_n = n$ one can think of $[X_{i,j}^{(n)}]$ as the incidence matrix of a random directed graph G_n on n vertices. Letting H be a fixed graph, the authors compute a formula for the expected number of graphs which are isomorphic to H and which are subgraphs of G_n. In the case where H is of order 3, a formula for the variance is also given. Some other characteristics are also studied.

Interpreting $\mathbb{X}_n := [X_{i,j}^{(n)}]$ as a matrix in the two-element algebraic field $\{0,1\}$, let $\mathcal{N}(\mathbb{X}_n)$ be the number of nonzero solutions x of the linear equation $\mathbb{X}_n^T x = 0$. A formula for $\mathbb{E}\mathcal{N}(\mathbb{X}_n)$ is given, and it is remarked that the expected total number of hypercycles of G_n is simply expressed in terms of this quantity.

Some remarks are made for more general exchangeable graphs. The formulas derived in the paper are made explicit in the case where $\pi_n(\theta) \propto \theta^{-\beta}1(\theta > \alpha/n)$

for some $\beta > 1$ and $0 < \alpha < n$.

In connection to the paper under review, the reader should be aware of the recent fundamental paper by Persi Diaconis and Svante Janson, “graph limits and exchangeable random graphs”, *Rendiconti di Matematica*, **28** (2008).