This is a review submitted to Mathematical Reviews/MathSciNet.

Reviewer Name: Konstantopoulos, Takis

Mathematical Reviews/MathSciNet Reviewer Number: 68397

Address:
Department of Mathematics
Uppsala University
PO Box 480
SE-75106 Uppsala
SWEDEN
takis@math.uu.se

Author: Blank, Michael

Title: Stochastic stability of traffic maps.

MR Number: MR2997699

Primary classification:

Secondary classification(s):

Review text:

A countable collection of balls (particles) of radius r are arranged so that their centers lie on a straight line identified with the set of real numbers $\mathbb{R}$. At each "time" $t \in \mathbb{Z}_+$, and for each ball (centered at x_i^t), toss a coin which shows heads with probability p, independently from ball to ball. If heads show up, shift the ball to the right by a given amount $v > 0$ unless the ball collides with the ball immediately to its right (in which case move x_i^t to $x_{i+1}^t - 2r$.) This defines a discrete-time Markov process $\pi(p, v, r, \mathbb{R})$ in an uncountable configuration space X.

The goal of this paper is to study existence and properties of stationary measures of the process.

Notice that, depending on the initial configuration, the radius r, and the "velocity" v, the process may live in a lattice, say, $\mathbb{Z}$, in which case it is denoted by $\pi(p, v, r, \mathbb{Z})$. This is, e.g., the case when $v = 1$ and $r = 1/2$.

The first result is that $\pi(p = 1, v = 1, r = 1/2, \mathbb{Z})$ has a nontrivial spatially-Markovian invariant measure μ which can be expressed as the weighted sum of two maximal-entropy measures for shift maps acting in opposite directions. Moreover, there is a 1-parameter family μ_ρ of invariant measures, where ρ is interpreted as spatial particle density. This is proved by employing connections to topological Markov chains. This result implies a corresponding result for the existence of a 1-parameter family μ_ρ of invariant measures for the $\pi(p = 1, v, r, \mathbb{R})$ process.

The second set of results concern existence of a 1-parameter family μ_ρ^p of invariant measures for $\pi(p, v, r, \mathbb{R})$ with $0 < p < 1$. Again, the case $\pi(p, v = 1, r = 1/2, \mathbb{Z})$ is first studied. Stochastic stability (a rather unfortunate term because different people mean different things by it) refers to continuity of μ_ρ^p as a function of p. It is shown that μ_ρ^p converges weakly to μ_ρ^1, as $p \to 1$.

Interestingly, the average velocity of particles (defined as the limit, as $t \to \infty$, of the position of a particle at time t divided by t) for the μ_ρ^p-stationary version of $\pi(p, v, r, \mathbb{R})$ admits an explicit formula as a function of p, v, r and ρ.