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Title: Markov dynamics on the Thoma cone: a model of time-dependent determinantal processes with infinitely many particles.

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The paper constructs a family $\{X^{(z,z')}\}$ of diffusions whose infinitesimal generator $\mathfrak{D}^{(z,z')}$ is obtained by a formal analytic extension of the Laguerre operator D_N acting on functions on N variables and having parameter $b > 0$. For $N = 1$, the Laguerre operator D is given by $D = x\frac{d^2}{dx^2} + (b - x)\frac{d}{dx}$ and generates a diffusion in $[0, \infty)$. The diffusion admits a unique stationary distribution with density $w(x) = x^{b-1}e^{-x}$ which also serves as a symmetrizing measure for D, i.e., D is a symmetric operator with respect to w. The eigenfunctions of D are the Laguerre polynomials which are pairwise orthogonal with respect to w. The extension of D to D_N has been described in the second author's earlier work [Olshanski 2011, Olshanski 2012]. To obtain $\mathfrak{D}^{(z,z')}$ from D_N, one rewrites D_N in terms of N-variate symmetric polynomials by a formal analytic extension (roughly speaking, by mapping the parameters N and b to a pair (z, z') of complex numbers). The thus-defined operator $\mathfrak{D}^{(z,z')}$ acts on the algebra Sym of all symmetric functions, and admits the representation of equation (1.1) in the paper under review. Whereas D_N is diagonalized by symmetric N-variable extensions of the Laguerre polynomials, $\mathfrak{D}^{(z,z')}$ is diagonalized by the so-called Laguerre symmetric functions $\mathfrak{L}_\nu^{(z,z')}$ indexed by integer partitions $\nu = (\nu_1, \nu_2, \dots)$, i.e., sequences of integers such that $\nu_1 \geq \nu_2 \geq \cdots \geq 0$ which are eventually equal to zero. The space Sym is mapped onto the so-called Thoma cone $\widetilde{\Omega}$, i.e., the set of triples (α, β, δ) where α, β are integer partitions and δ a positive number, such that $\sum_i \alpha_i + \sum_i \beta_i \leq \delta$. This is inspired by the theory of representations of the infinite symmetric group $S(\infty)$: [Thoma 1964] obtained that the extremal characters of $S(\infty)$ are in one-to-one correspondence with pairs (α, β) such that $(\alpha, \beta, \delta = 1) \in \widetilde{\Omega}$. In this way, the authors view the operator $\mathfrak{D}^{(z,z')}$ as acting on a certain class of functions on $\widetilde{\Omega}$. The main results of the paper are: (i) $\mathfrak{D}^{(z,z')}$ is dissipative and is the generator of a Feller semigroup $T^{(z,z')}(t)$, $t \geq 0$, admitting a unique stationary distribution. (ii) The corresponding diffusion process $X^{(z,z')}$ is a time-dependent determinantal point process whose transition probability kernel is the so-called Whittaker kernel which appeared in [Borodin and Olshanski 2006]. The construction uses techniques from the theory of weak convergence of Markov processes [Ethier and Kurtz 1986].

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