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Short title: Ergodicity of a stress release point process seismic model with aftershocks.

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Review text:

amsmath,amsfonts

This paper considers the dynamics of a point process N on the real line defined through its stochastic intensity λ as follows:

$$\lambda(t) = \varphi(X(t)) + Y(t),$$

where $\varphi : \mathbb{R} \to \mathbb{R}_+$ is nondecreasing with $\lim_{x\to-\infty} \varphi(x) = 0$, $\lim_{x\to\infty} \varphi(x) = \infty$, and

$$X(t) = X_0 + ct - \sum_{n=1}^{N(t)} Z_n,$$
$$Y(t) = Y_0 e^{-\alpha t} + k \int_{(0,t]} e^{-\alpha(t-s)} dN(s), \quad t \geq 0.$$

The only randomness in the system is through the random variables $(Z_n, n \in \mathbb{N})$, assumed to be independent, identically distributed, nonnegative, and with finite mean. The constants c, k, α are positive. $N(t)$ represents the number of points on the interval $(0, t]$ and is itself defined by requiring that it have stochastic intensity $\lambda(t)$, namely that $N(t) - \int_0^t \lambda(s)ds$, $t \geq 0$, is a martingale with respect to a natural filtration $(\mathcal{F}_t, t \geq 0)$ [see Brémaud (1981)]. It is observed that (X, Y) is a Markov process in the plane. The authors are concerned with the "ergodicity" of the system, interpreted as stability of the above Markov process under the condition $k < \alpha$. This is done by first proving the stability of an embedded Markov chain $((X(t_n), Y(t_n)), n \in \mathbb{Z}_+)$, where $0 = t_0 < t_1 < \cdots$ are the points of N. The technique for this is Foster's criterion requiring the construction of an appropriate Lyapunov function (a piecewise linear function on the plane is chosen here). Subsequently, the discrete-time result is transferred to the continuous time process (X, Y). It is shown that (X, Y) admits a unique stationary and ergodic version and that, starting from arbitrary initial condition (X_0, Y_0), we have convergence to the stationary version in total variation norm. To obtain this result an additional assumption is needed, namely that the distribution of Z_1 have an absolutely continuous component. The paper is motivated by stochastic models of earthquakes. It extends results by Last (2004). There is a typo in the paper: the formula in the middle of p. 391 should read:

$$N(t) = \int_{(0,t]} \int_{\mathbb{R}} \mathbf{I}(z \le \lambda(s-)) \, \Pi(ds \times dz).$$

Bibliography used in this review:

Brémaud, Pierre. Point processes and queues. Martingale dynamics. Springer Series in Statistics. Springer-Verlag, New York-Berlin, 1981. xviii+354 pp. ISBN: 0-387-90536-7 MR0636252 (82m:60058)

Last, Günter. Ergodicity properties of stress release, repairable system and workload models. Adv. in Appl. Probab. 36 (2004), no. 2, 471–498. MR2058146 (2005b:60194)