This is a review submitted to Mathematical Reviews/MathSciNet.

Reviewer Name: Konstantopoulos, Takis

Mathematical Reviews/MathSciNet Reviewer Number: 68397

Address:
Department of Mathematics
Uppsala University
PO Box 480
SE-75106 Uppsala
SWEDEN
takiskonst@gmail.com

Author: Brudern, Jorg; Dietmann, Rainer

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amsmath,amsthm,amsfonts,amssymb,mathrsfs,epsfig,bm

Given a set of coefficients $\mathbf{a} = \{a_{j_1,\dots,j_d}\}$, let Let $F_{\mathbf{a}}(X_1,\dots,X_s)$ be a homogeneous polynomial in s variables of degree d, that is,

$$F_{\mathbf{a}}(X_1,\dots,X_s) = \sum_{1 \le j_1 \le \cdots \le j_d \le s} a_{j_1,\dots,j_d} X_{j_1} \cdots X_{j_d}.$$

Let $q > 1$ be a positive integer, $1 \le B < q$, and let $X(q,B)$ be the set of all integer s-tuples $(x_1,\dots,x_s)$ such that $-B \le x_j \le B$ and $\gcd(x_j,q) = 1$ for all j. Let $N_{\mathbf{a}}(q,B)$ be the number of $(x_1,\dots,x_s) \in X(q,B)$ that satisfy

$$F_{\mathbf{a}}(x_1,\dots,x_s) \equiv 0 \mod q.$$

A set $\mathbf{a}$ of coefficients is called admissible if the coefficients range over a complete set of residues (mod q) or some of the diagonal coefficients $a_{j,\dots,j}$ are set equal to 0. Let $\mathfrak{A}$ be the set of admissible coefficients. Then

$$\sum_{\mathbf{a} \in \mathfrak{A}} N_{\mathbf{a}}(q,B) = \frac{|\mathfrak{A}|\,|X(q,B)|}{q}.$$

It makes sense then to consider

$$V = \sum_{\mathbf{a} \in \mathfrak{A}} \left(N_{\mathbf{a}}(q,B) - \frac{|X(q,B)|}{q} \right)^2$$

as a "variance" (that is, $V/|\mathfrak{A}|$ is the variance of the random variable $N_{\mathbf{a}}(q,B)$ when $\mathbf{a}$ is chosen uniformly at random from $\mathfrak{A}$). The main result is the following bound for V. If $s \geq 3$ and $0 < \delta \leq 1$, there exists constant C such that if q contains no prime factors less than q^δ and $q^{1/s} \leq B < q$ then

$$V \leq \frac{C|\mathfrak{A}|}{q^2}(B^s q + B^{2s} q^{\delta(2-s)}).$$

An application to the geometry of numbers is also given.

Comments to the MR Editors (not part of the Review Text):
The paper requires the use of a curly X. I used the command X It didn't work on your system. I indicated the packages I use, the one responsible for this is mathrsfs. Still, it didn't work.