This is a review submitted to Mathematical Reviews/MathSciNet.

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Title: Elements related to the largest complete excursion of a reflected BM stopped at a fixed time. Application to local score.

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Primary classification:

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Review text:

For a reflected Brownian motion $U(t) = B(t) - \inf_{0\le s\le t} B(s)$, where $B(t)$, $t \ge 0$, is a standard Brownian motion, consider all complete excursions before time t, i.e., all pieces $(U(s) : g \le s \le d)$ of $(U(s) : 0 \le s \le t)$ such that $U(g) = U(d) = 0$, $U(s) > 0$ for all $g < s < d$, and $0 \le g < d \le t$. The authors focus on the highest excursion i.e. the excursion for which $\sup_{g<s<d} U(s)$ is largest. Let $U^*(t)$ be the height of the highest excursion, and let $\theta^*(t)$ be the time elapsed from the beginning of the excursion until the maximum is achieved. The authors derive expressions for the joint density of $(U^*(t), \theta^*(t))$ in terms of random variables with known densities. The main tool is excursion theory. Although the expressions are not explicit, they are amenable to simulation. The marginal densities are also computed.

The motivation comes from a discrete-time setup, namely a sequence ϵ_n of zero-mean unit-variance random variables, with partial sums $S_n = \epsilon_1 + \cdots + \epsilon_n$ which are reflected according to $U_n = S_n - \min_{i\le n} S_i$. Letting U_n^*, θ_n^* be the analogs of $U^*(t)$, $\theta^*(t)$, it is formally proved that $n^{-1/2}(U_n^*, \theta_n^*)$ converges in distribution to $(U^*(1), \theta^*(1))$ as $n \to \infty$. The discrete-time setup is motivated by applications in molecular biology.