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Title: Optimal concentration inequalities for dynamical systems.

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The paper proves concentration inequalities for dynamical systems of various types. A stationary sequence $(Z_0, Z_1, \dots)$ of random variables (with values in a metric space) are said to satisfy a concentration inequality if there is a constant C, such that, for all n, and for any function $K : X^n \to \mathbb{R}$ which is Lipschitz in each of its arguments, we have

$$\mathbb{E}\big[\exp\{K(Z_0, \dots, Z_n) - \mathbb{E}K(Z_0, \dots, Z_n)\}\big] \le \exp C(L_0^2 + \cdots + L_{n-1}^2),$$

where L_i is the Lipschitz constant with respect to the i-th variable of K. The inequality immediately implies, using Markov's inequality, that $K(Z_0, \dots, Z_{n-1})$ concentrates around its mean exponentially fast (in fact like $c_1 e^{-c_2 t^2}$). In some cases, the "exponentially fast" must replaced by "polynomially fast", giving polynomial concentration inequalities.

The point of view in this paper is that of a stationary dynamical system (X, T, μ), where $Z_n(x) = T^n x$ on a metric space X, and specializes to cases where X is a set of sequences.

In the first part of the paper, the authors consider subshifts of finite type (X, T, μ) where X is a subset a sequence space (with values in a finite alphabet), T the natural shift, and μ a Gibbs invariant measure. Both one- and two-sided cases, are considered. It is shown that, in both cases, exponential concentration inequalities hold. Then, the so-called uniform Young towers [You98] and [You99] with exponential tails are studied. Again, these dynamical systems are shown to satisfy exponential concentration inequalities. Non-uniform Young towers with polynomial tails (i.e., where the so-called return-time function possesses polyno-

mial moments, with respect to the invariant measure μ, of all orders at least 2) are also studied and are shown to satisfy polynomial concentration inequalities. These inequalities are optimal. A number of special, and important, dynamical systems are also studied. The proofs rely heavily on martingale techniques.