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The paper is motivated by a problem in dynamical systems: for an ergodic transformation on a probability space $(X, \mathcal{B}, m)$ and a probability measure μ on the integers $\mathbb{Z}$, define the random measure $\widetilde{\mu}(x, B) := \sum_{k\in\mathbb{Z}} \mu(k)\mathbf{1}_{\tau^k x\in B}$, $B \in \mathcal{B}$. (I am using a slightly different notation from that of the paper in order to make my point.) Consider then an m-integrable random variable $f : X \to \mathbb{R}$ and let $\widetilde{\mu}^n f(x) := \int_X f(y)\widetilde{\mu}^n(x, dy)$, where $\widetilde{\mu}^n(x, \cdot) := \widetilde{\mu}^{n-1}(x, \cdot) * \widetilde{\mu}(x, \cdot)$ (where $*$ denotes convolution). The problem is the a.e. convergence (or failure of it) of $\widetilde{\mu}^n f$, as $n \to \infty$. It is known that boundedness/unboundedness of the angular ratio of μ plays an important role in this problem. A strictly aperiodic μ has bounded angular ratio iff $\sup |\widehat{\mu}(\lambda) - 1|/(1 - |\widehat{\mu}(\lambda)|) < \infty$, where $\widehat{\mu}(\lambda) := \sum_k \lambda^k \mu(k)$, $\lambda \in \mathbb{C}$, and where the sup is taken over the unit disk except the point $\lambda = 1$. The author deals with properties of the angular ratio of μ in a more general setup, looking at random variables X and the associated function

$$R_\varphi(t, \alpha) := |\operatorname{Im} \varphi(t)|/(1 - \operatorname{Re} \varphi(t))^\alpha, \quad t \in \mathbb{R},$$

for $1/2 \le \alpha \le 1$, where $\varphi(t) = \mathbb{E}e^{itX}$ is the characteristic function of X. Bellow, Jones and Rosenblatt proved that if $\mathbb{E}|X| < \infty$ but $\mathbb{E}X \neq 0$ then $\lim_{t\to 0} R_\varphi(t, 1) = \infty$ and constructed a random variable X with $\mathbb{E}X = 0$ for which the same thing holds. Ostrovskii proved that there exist random variables X with $\mathbb{E}|X|^{1+\epsilon} < \infty$ for some small $\epsilon > 0$ and $\mathbb{E}X = 0$ for which $\limsup_{t\to 0} R_\varphi(t, 1) = \infty$ and conjectured that it is possible to have $\liminf_{t\to 0} R_\varphi(t, 1) < \infty$. The paper deals with these problems by constructing examples and by giving necessary and sufficient conditions for $\lim_{t\to 0} R_\varphi(t, \alpha) = \infty$, formulated in terms of the distribution function of X and truncated moments of it.