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Consider the nonlinear control system

$$\dot{x} = f(x, u)$$

in $\mathbb{R}^d$, with controls u ranging in a set $\mathcal{U}$ of locally integrable functions, and assume everything one could wish for: that $f(\cdot, u)$ is Lipschitz continuous; that for each initial state x_0 there is a unique solution $\varphi(t, x_0, u)$, $t \geq 0$, depending continuously on x_0; and that for each control u there is a unique stable equilibrium which, without loss of generality, may be taken to be the origin of $\mathbb{R}^d$.

Such a system is called exponentially controllable to the equilibrium if for each compact neighborhood K of the equilibrium there are constants M, α such that for all initial states $x_0 \in K$ there is a control u such that

$$\|\varphi(t, x_0, u)\| \leq M e^{-\alpha t} \|x_0\|, \quad t \geq 0.$$

The question dealt with in this paper is whether we can achieve exponential controllability with finitely many controls and, if yes, how efficiently. It is easily seen that for the nicest systems (linear systems, for example), this is not possible. However, weakening the last display to

$$\|\varphi(t, x_0, u)\| \leq e^{-\alpha t}(\varepsilon + M\|x_0\|), \quad 0 \leq t \leq T,$$

may work. The cardinality of the smallest finite subset $\mathcal{S}$ of $\mathcal{U}$ for which this holds is denoted by $s_{\text{stab}}(T, \varepsilon, \alpha, M, K)$. The stabilization entropy is then defined as

$$h_{\mathrm{stab}}(\alpha, M, K) := \lim_{\varepsilon \to 0} \overline{\lim}_{T\to\infty} \frac{1}{T} \log s_{\mathrm{stab}}(T, \varepsilon, \alpha, M, K).$$

A large part of the paper is devoted to obtaining upper and lower bounds for this quantity. For linear systems, bounds are expressed in terms of the real parts of the eigenvalues of the system matrix.

Another way to measure efficiency is by replacing the ε above by a function $\gamma(t)$ which may decrease slower than exponentially in a neighborhood of the equilibrium. If, for each compact neighborhood K of the equilibrium, we let $\mathcal{R}(\gamma, \varepsilon, \alpha, M)$ be a subset of $\mathcal{U}$ such that, for each $x_0 \in K$, there is $u \in \mathcal{R}(\gamma, \varepsilon, \alpha, M)$ achieving

$$\|\varphi(t, x_0, u)\| \leq \gamma(t) + Me^{-\alpha t}\|x_0\|, \quad t \geq 0,$$

we can define a minimal bit rate b_{stab} for stabilization in a natural way. Bounds for b_{stab} are derived. Finally, inequalities relating the quantities h_{stab} and b_{stab} are also found.