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Consider a Lévy noise Π on $\mathbb{R}^d$, that is, a collection of random variables $\Pi(B)$, indexed by the Borel sets B of $\mathbb{R}^d$, such that, if $B_1, \ldots, B_n$ are pairwise disjoint Borel sets then $\Pi(\cup_{i=1}^n B_i) = \sum_{i=1}^n \Pi(B_i)$, a.s., and the random variables in the sum are independent; also, the law of $\Pi(B)$ depends on B only through its Lebesgue measure. For example, Π can be a Gaussian white noise or a Poisson process.

If G is a group of measurable transformations on $\mathbb{R}^d$ (with measurable inverses) then the G-invariant σ-field $\mathcal{I}_G$ is defined to contain all events A such that $\{\Pi \in A\} = \{\Pi \circ g^{-1} \in A\}$, a.s. Suppose also that the law of Π is invariant under the actions of G. The question is to identify conditions under which the events in the σ-algebra $\mathcal{I}_G$ are trivial (i.e., they have probability 0 or 1). It is known that if G contains all measurable bijections preserving the Lebesgue measure then the answer is affirmative: all events in $\mathcal{I}_G$ are trivial. Note that if G_1 is a subgroup of G_2 then the G_1-invariant σ-field contains the G_2-invariant σ-field.

Let Γ be the subgroup of the group of measure-preserving bijections consisting of linear bijections of $\mathbb{R}^d$. Let G be a closed subgroup of Γ. The main theorem of the paper is: $\mathcal{I}_G$ is trivial if and only if G is not compact.

A main ingredient of the proof is a result [Merzljakov (1966)] asserting that if a subgroup of Γ is closed but not compact then there exists $g \in G$ such that the cyclic subgroup generated by g and g^{-1} is not relatively compact. The author remarks that this is a non-trivial statement, related to the so-called Auerbach problem.

One motivation [Holroyd *et al.* (2009)] for it is a recent result stating the following: Take Π to be a homogeneous Poisson process on $\mathbb{R}^d$ with intensity $\beta > 0$. Let $\alpha \in (0, \beta)$. Then there exists a deterministic measurable function Θ, mapping point processes into point processes, such that, (i) a.s., $\Theta(\Pi) \leq \Pi$ (i.e. $\Theta(\Pi)$ has fewer points than Π; it is, in other words, a thinning of Π), (ii) $\Theta(\Pi)$ is also Poisson process but with the smaller intensity α, and (iii) Θ is invariant under the group of affine Euclidean isometries (rigid body motions). The question is whether there exists a map satisfying (i), (ii) and (iii) under some larger group of measure preserving affine maps. The result of this paper shows that this is not possible.

Another motivation [Kallenberg (1977), Mecke (1979)] comes from a de Finetti representation of the law $\mathbb{P}$ of a simple point process on $\mathbb{R}^d$: if $\mathbb{P}$ is invariant under the actions of the group G of measure-preserving bijections then $\mathbb{P}$ is a mixture of (laws of) Poisson processes. It is known that if $\mathbb{P}$ is invariant under the actions of the smaller group of affine maps preserving Lebesgue measure then the conclusion is false.

The paper is very interesting and very carefully written .

Bibliography used in this review:

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