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Title: Global asymptotic stabilization of nonlinear deterministic systems using Wiener processes.

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Review text:

The paper deals with the design of a noise-assisted stabilizing controller for a nonlinear finite-dimensional deterministic system. It is known that a deterministic system of the form

$$\dot{x} = f(x) + g_1(x)u_1 + \cdots + g_m(x)u_m,$$

where the state x evolves in $\mathbb{R}^n$, the $f(x)$, $g_i(x)$ are smooth vector fields on $\mathbb{R}^n$, and $u_1, \ldots, u_m$ are scalar controls, may not be asymptotically stable. The authors investigate when a controller of the form

$$u_i dt = v_i(x)dt + B_i(x) \circ dw, \quad i = 1, \ldots, n,$$

where the $v_i(x)$ and $B_i(x)$ are scalar functions and w a one-dimensional Brownian motion (Wiener process), stabilize the system. The controller should be interpreted as a stochastic differential equation in Stratonovich form and so $\int_0^t B_i(x(s)) \circ dw(s)$ is a Stratonovich integral. Combining the two displays one obtains the closed-loop system, in Stratonovich form. The authors make use of a known result (by Khasminskii and Kushner, stating that the system is stable [specifically, globally asymptotically stable in probability] provided that a global Lyapunov function, satisfying some strong properties, exists), in order to prove a sufficient condition for the stability of the system at hand. The condition involves the existence of an appropriate Lyapunov function, called stochastic control Lyapunov function (SCLF) in this paper. A 3-dimensional simple example (the so-called Brockett integrator) is also studied, largely numerically. The proposed SCLF is quite complicated but it works.