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Title: Subexponential asymptotics of the stationary distributions of GI/G/1-type Markov chains.

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The stationary distribution of a certain Markov chain (X_n, S_n), $n \in \mathbb{Z}_+$, with values in $\mathbb{Z}_+ \times \{1, \ldots, M\}$ is considered and "subexponential" asymptotics are derived. The Markov chain arises in queueing theory. The first component typically describes the size of the queue at embedded epochs; and the second component is an auxiliary random variable related to the stage of the service. The transition probability matrix T of the chain can best be visualized as a matrix indexed by $\mathbb{Z}_+ \times \mathbb{Z}_+$ whose entries $T_{i,j}$ are $M \times M$ smaller matrices (with M possibly changing at the boundary). Also, the matrix has a certain homogeneity in that $T_{i,j}$ depends on the subscripts only through $j - i$ (away from the boundary); write $T_{i,j} = A(j - i)$. Assuming that $A(k)$ behaves, as $k \to \infty$, like $\mathbb{P}(Y > k)$, where Y is a finite-mean positive random variable, it is shown that the tail of the stationary distribution behaves like $\mathbb{P}(Y_e > k)$, where Y_e is the random variable which makes the random measure $N(\cdot) := \sum_{n=0}^{\infty} 1(Y_e + Y_1 + \cdots + Y_n \in \cdot)$ have a distribution which is invariant under time-shifts (here, $Y_1, Y_2, \ldots$ are i.i.d. copies of Y and independent of Y_e). In other words, $\mathbb{P}(Y_e = k) = \mathbb{P}(Y > k)/\mathbb{E}Y$. Such are the results of this paper. According to the abstract,

> subexponential asymptotics are considered in two cases: (i) the phase transition matrix in non-boundary levels is stochastic; and (ii) it is strictly substochastic. For case (i), we present a weaker sufficient condition for the subexponential asymptotics than those

> given in the literature. As for case (ii), the subexponential asymptotics has not been studied, as far as we know. We show that the subexponential asymptotics in case (ii) is different from that in case (i). We also study the locally subexponential asymptotics of the stationary distributions in both cases (i) and (ii).

The paper is written with the mind that the reader knows all conventions of the specific type of queues the authors study. The way that the matrix is obtained from the queueing system is not described in the paper. Moreover, the 3- or 4-letter acronyms BMAP, SMAP, MSP appearing in the first paragraph are not defined. This is a drawback for a paper published in a journal titled "stochastic models" which, apparently, is not specialized in queueing theory only.