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Title: Moments of discrete measures with dense jumps induced by β-expansions.

MR Number: MR2993831

Primary classification:

Secondary classification(s):

Review text:

For $\beta > 1$, define the function

$$\mu_\beta(x) := \sum_{n=0}^{\infty} \frac{\lceil (n+1)x \rceil - \lceil nx \rceil}{\beta^{n+1}}, \quad 0 \leq x \leq 1.$$

The author studies several properties of this function. In particular, μ_β is strictly increasing, and is the left-continuous distribution function of the measure

$$\sum_{r \in \mathbb{Q} \cap [0,1)} \frac{\beta - 1}{\beta(\beta^{q(r)} - 1)} \, \delta_r,$$

where $\mathbb{Q}$ is the set of rational numbers and $q(r) = q$ if $r = p/q$ with p, q coprime integers. The moments $M_m := \int x^m \, d\mu_\beta(x)$ are expressed in terms of the Bernoulli numbers and polylogarithm functions. Asymptotics of M_m as $m \to \infty$ are derived in explicit form due to the fact that the exponential generating function $\int e^{sx} \, d\mu_\beta(x)$ has a nice expression.