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Title: On the Gaussian limiting distribution of lattice points in a parallelepiped.

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This work is motivated by a theorem by J. Beck (1992). Let X, Y, Z are independent random variables, all uniformly distributed between 0 and 1. Consider the lattice $Z + n\sqrt{2}$, $n \in \mathbb{Z}$, and look at the fractional parts $\{n\sqrt{2} + Z\}$ of its points. Count the number of these fractional parts, for $0 \le n \le Nx$, for positive integer N, that fall between 0 and Y and subtract its "mean" value NXY. Then

$$(\log N)^{-1/2}\Bigg(\sum_{n=0}^{[NX]} \mathbf{1}_{\{n\sqrt{2}+Z\}\le Y} - NXY\Bigg)$$

converges in distibution, as $N \to \infty$, to a normal random variable with zero mean and positive variance.

The main result of this paper is a far-reaching generalization of the theorem in s dimensions. Let Γ be a lattice obtained from a module in a totally real algebraic number field. Denote by $\det \Gamma$ the volume (s-dimensional Lebesgue measure) of the fundamental region of Γ. For a bounded set $\mathcal{O}$ with volume $\operatorname{vol} \mathcal{O}$ and a vector $\mathbf{x} \in [0,1]^s$ let

$$\mathcal{R}(\mathcal{O} + \mathbf{x}, \Gamma) := \sum_{\gamma \in \Gamma} \mathbf{1}_{\gamma \in \mathcal{O}+\mathbf{x}} - \frac{\operatorname{vol} \mathcal{O}}{\det \Gamma}.$$

Let $\mathbb{K}_s := [-1/2, 1/2]^s$, $\mathbf{N} = (N_1, \ldots, N_s)$ an integer vector, and $\mathbf{\Theta}$ be a random variable uniformly distributed in $[0,1]^s$. Consider the set

$$\mathbf{\Theta} \cdot \mathbf{N} \cdot \mathbb{K}_s := \{(\Theta_1 N_1 y_1, \ldots, \Theta_s N_s y_s) : (y_1, \ldots, y_s) \in \mathbb{K}_s\},$$

and let

$$w(\mathbf{N},\mathbf{x})^2 := \mathrm{Var}[\mathcal{R}(\mathbf{\Theta}\cdot\mathbf{N}\cdot\mathbb{K}_s+\mathbf{x},\Gamma)].$$

Then $w(\mathbf{N},\mathbf{x})$ is finite and positive and

$$\frac{\mathcal{R}(\mathbf{\Theta}\cdot\mathbf{N}\cdot\mathbb{K}_s+\mathbf{x},\Gamma)}{w(\mathbf{N},\mathbf{x})}$$

converges in distribution to a standard normal random variable for all $\mathbf{x}\in[0,1]^s$. Moreover the uniform deviation of the distribution function of this ratio from the standard normal distribution function is of order $(\log_2(N_1\cdots N_s))^{-1/15}$.

According to the author, the problems considered in the paper are related to quantum chaos theory as the number of eigenvalues of a quantum mechanical operator in a large interval $[0,t]$ leads to the problem of couting lattice points in a domain dilated by the factor t.