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Title: Central limit theorem for $\mathbb{Z}_+^d$-actions by toral endomorphisms.

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Review text:

The paper deals with a multidimensional central limit theorem for the following situation. Let $A_1, \ldots, A_d$ be integer $s \times s$ matrices which commute with one another and set $A := (A_1, \ldots, A_d)$. For integers $n_1, \ldots, n_d$ set $n := (n_1, \ldots, n_d)$ and $A^n := A_1^{n_1} \cdots A_d^{n_d}$. Let $f : \mathbb{R}^s \to \mathbb{R}$ be a locally integrable 1-periodic function with zero mean and set

$$S_N(f) := \sum_{n \leq N} f(A^n x),$$

where x is a random variable, uniformly distributed in the unit cube $[0,1]^s$. The paper considers the asymptotic behavior of $S_N(f)$ and, also, the joint asymptotic behavior of the random q-tuple

$$S_{N_1}(f), \ldots, S_{N_q}(f),$$

as $N_1 = (N_{11}, \ldots, N_{1d}), \ldots, N_q = (N_{q1}, \ldots, N_{qd})$ tend to infinity. Let $\sigma(f)^2$ be the asymptotic variance of $S_N(f)$ (defined in the most natural way) and assume that it is finite. One of the main theorems then states that

$$\frac{1}{\sigma(f)} \left(\frac{S_{N_1}(f)}{\sqrt{N_{11} \cdots N_{1d}}}, \ldots, \frac{S_{N_q}(f)}{\sqrt{N_{q1} \cdots N_{qd}}} \right)$$

converges in distribution to a standard Gaussian law in $\mathbb{R}^q$, as $\min_{i,j} N_{ij} \to \infty$, provided that the eigenvalues of A^n are never roots of unity, for any $n \in \mathbb{Z}^d \setminus \{0\}$. A functional central limit theorem and an almost sure central limit theorem are also proved.