This is a review text file submitted electronically to MR.

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Short title: A note on diffusion limits of chaotic skew-product flows.

MR Number: 2776125

Primary classification: 37A50

Secondary classification(s): 60H10, 34E13, 60F05, 60F10

Review text:

This nice short paper gives a rigorous demonstration of convergence of an ODE to an SDE. Roughly speaking, it gives the essential reason of how randomness emerges from determinism and shows that, mathematically, there is nothing surprising once the right assumptions are made (once the deterministic system exhibits the right kind of behavior).

Specifically, consider the autonomous multidimensional ODE

$$\dot{y}^{(\epsilon)} = \epsilon^{-2} g(y^{(\epsilon)}),$$

where ϵ is a small positive parameter, and assume that it has a compact attractor Λ supporting an invariant probability measure μ. There are many examples, some of which mentioned in the paper, exhibiting this kind of behavior. This ODE drives

$$\dot{x}^{(\epsilon)} = \epsilon^{-1} f_0(x^{(\epsilon)}) + f(x^{(\epsilon)}, y^{(\epsilon)}),$$

in some other Euclidean space. Assuming (i) Lipschitz conditions for the functions involved, (ii) that the sequence

$$W_n(t) := \frac{1}{\sqrt{n}} \int_0^{nt} f_0(y^{(1)}(\tau)) d\tau$$

converges weakly (in the space of probability measures on continuous functions equipped with the topology of local uniform convergence) to a Brownian motion $\sqrt{\Sigma} W$ (with covariance matrix depending on g and f_0), and (iii) that a large deviations principle holds, it is shown that $x^{(\epsilon)}$ converges, as $\epsilon \downarrow 0$, to a stochastic process X solving the SDE

$$dX = F(X)dt + \sqrt{\Sigma}dW,$$

where $F(x) = \int_\Lambda f(x,y)\mu(dy)$. It is seen from the literature that the asumption that W_n converges to a Brownian motion is very natural in many classes of systems. The paper provides a very good, and to the point, demonstration of how this assumption leads to the conclusion that the deterministic functions $x^{(\epsilon)}$ converge to the random function X.