This is a review submitted to Mathematical Reviews/MathSciNet.

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This paper is motivated by the recent series of works on existence of equivariant operations on Poisson processes such as thinning, allocation and matching. For example, a matching between two independent Poisson processes is a bijection between their points. Isometry-equivariant matchings have been constructed by Holroyd, Pemantle, Peres and Schramm (2009) and Holroyd (2011).

The problem is formulated in the language of ergodic theory. On some σ-finite measure space $(X, \mathcal{B}, \mu)$, with $\mu(X) = \infty$, a μ-preserving transformation $T : X \to X$ is given. Let μ^* be the law of a Poisson process on X with intensity measure μ. Thus, if X^* is the set of countable subsets of X equipped with a natural σ-algebra $\mathcal{B}^*$, then $(X^*, \mathcal{B}^*, \mu^*)$ is a probability space–the "Poisson space". Let $T_* : X^* \to X^*$ be the natural lifting of T from X to X^*, i.e., if $\omega \in X^*$ then $T_*\omega$ is the collection $T(x)$ for all points $x \in \omega$. Assume further that T is conservative, meaning that for any set $W \in \mathcal{B}$ with $\mu(W) > 0$ its transformations $\{T^{-n}W, n = 0, 1, \dots\}$ are μ-a.e. pairwise disjoint. Finally, let $T \times T_*$ be the product of the two maps T and T_* (Poisson product). Following a condition for ergodicity of products of maps by Michael Keane, the first theorem proved is that, under the above conditions, T is ergodic if and only if $T \times T_*$ is ergodic.

This result is used to show that, under the condition that T is measure preserving and ergodic, it is impossible to find (nontrivial) T-equivariant Poisson thinnings, allocations or matchings. The word "equivariant" for one of these operations is taken to mean that the operation is compatible with T. For example, a (deterministic) thinning Ψ on X^* is a map from X^* into itself which selects a certain subset of X^*. The thinning Ψ is equivariant if $\Psi \circ T_* = T_* \circ \Psi$.

The next part of the paper is devoted to the study of a certain operation on Poisson processes on $X = \mathbb{R}_+$, with μ =Lebesgue, called "leftmost position transformation": given a μ-preserving T, let $\kappa(\omega)$ be the smallest number k of iterations required so that $T^k\omega$ has no points to the left of the smallest point of ω, and let $T_*\omega := T_*^{\kappa(\omega)}\omega$. It is proved that if T is conservative and ergodic then T_*^κ is ergodic.

The last section briefly addresses ergodicity of the Poisson product of measure preserving group transformations, such as the group of isometries on $\mathbb{R}^n$. The paper is very well written and is a nice addition to this rich area of current research interest.

Comments to the MR Editors (not part of the Review Text):

I am trying to change my email address from takis@math.uu.se to takiskonst@gmail.com because my university's mail system is ancient and can't use it. I tried but I'm not sure I succeeded. It still shows the incorrect email on my profile. Can you please help with this? Thanks. Takis Konstantopoulos