This is a review submitted to Mathematical Reviews/MathSciNet.

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Title: Entropy rate for hidden Markov chains with rare transitions.

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Review text:

Given a Markov chain (X_n) in a finite state space S with transition probability matrix $P(p) = I + pA$, where p is a small positive parameter, and a sequence (Y_n), with values in some set T, obtained by passing (X_n) through a homogeneous memoryless channel, this paper derives an estimate for the entropy $h(Y)$ of (Y_n). Specifically, letting Q_{ij} be the probability that the channel output is $j \in T$ given that the input is $i \in S$, and assuming that $P(p)$ is irreducible for all small p, and that the rows of the matrix Q are all distinct, it is shown that there exists a constant $C > 0$, such that

$$h(X) + h(Y|X) - Cp \le h(Y) \le h(X) + h(Y|X),$$

for all small $p > 0$. If π is the invariant probability distribution of X ($\pi A = 0$), then, as usual, $h(X) = -\sum_{i,j} \pi_i P_{ij} \log P_{ij}$, and $h(Y|X) = -\sum_{i,j} \pi_i Q_{ij} \log Q_{ij}$. The method of proof consists in showing that $H(X|Y) \le Cp$ by splitting the probability space into a suitable finite partition which takes into account the number of transitions that the chain X can perform in a stretch L of time which goes to ∞ appropriately as $p \to 0$. Essentially, the authors make direct use of the intuition that, since p is small, the chain will stay constant for long stretches of time, thereby allowing reconstruction of the input symbol from the observation of the output with error probability $O(p)$. It is also shown the upper bound can be improved, under the mild assumption that there exist $i, i' \in S$, $j \in T$, with $P_{ii'} > 0$, $Q_{ij} > 0$, and $Q_{i'j} > 0$. In this case, there is a constant

$c > 0$ such that

$$h(Y) \leq h(X) + h(Y|X) - cp.$$

The paper improves and generalizes the result of Nair *et. al* (2005), concerning an inequality with $O(p \log p)$ error for the case of some 2-state Markov chain and for the binary symmetric channel.

Bibliography used in this review:

- C. Nair, E. Ordentlich, and T. Weissman. Asymptotic filtering and entropy rate of a hidden Markov process in the rare transitions regime. In: *Proc. International Symposium on Information Theory*, pp. 1838-1842, 2005.