This is a review submitted to Mathematical Reviews/MathSciNet.

Reviewer Name: Konstantopoulos, Takis

Mathematical Reviews/MathSciNet Reviewer Number: 68397

Address:
Department of Mathematics
Uppsala University
PO Box 480
SE-75106 Uppsala
SWEDEN
takis@math.uu.se

Author: Pitman, Jim; Ross, Nathan

Title: The greatest convex minorant of Brownian motion, meander, and bridge.

MR Number: MR2948693

Primary classification:

Secondary classification(s):

Review text:

The greatest convex minorant of a given real valued function on a closed interval I is the largest convex function which is below the given function on I. Continuing work by Jim Pitman, in this article, the authors study the greatest convex minorant of a Brownian motion on an interval $[0, \Gamma_1]$, with Γ_1 an independent exponential random variable, and related stochastic processes. Two descriptions of this minorant are provided, one in terms of a Poisson point process and another in terms of a stochastic recursion.

Focusing on Brownian motion B on $[0, \Gamma_1]$, one observes that the greatest convex minorant is a piecewise linear process with finitely many segments on every open interval contained in $[0, \Gamma_1]$, and accumulation points at the ends. The first main theorem states that if we consider the set of points (x, s), where x is the length of a face and s its slope, then this set is a Poisson point process on $\mathbb{R}_+ \times \mathbb{R}$ with intensity

$$\frac{\exp(-x(2+s^2)/2)}{\sqrt{2\pi x}}.$$

Letting $0 < \cdots < \alpha_{-1} < \alpha_0 < \alpha_1 < \cdots < 1$ be the times of the breakpoints of the greatest convex minorant of a B on $[0, 1]$, where α_0 is the time of global minimum $M = B(\alpha_0) = \min_{0 \le t \le 1} B(t)$ of B on $[0, 1]$, a sequential description of these times, together with the slopes of the faces, is obtained by making use of Denisov's decomposition: conditional on α_0, the processes $B(\alpha_0 + u) - M$, $0 \le u \le 1 - \alpha_0$ and $B(\alpha_0 - u) - M$, $0 \le u \le \alpha_0$, are independent Brownian meanders of lengths $1 - \alpha_0$ and α_0.

A certain stochastic recursion, termed as (τ, ρ)-recursion is key to the sequential description of the greatest convex minorant. There are several interesting and non-obvious distributional properties associated with this recursion and which are proved in this paper by using the relation of the recursion to the process. (Independent proofs are not available.) A central limit theorem for the recursion is also obtain by making use of positive recurrent Harris chain theory.

In addition, there are results about the greatest convex minorant of a $\mathrm{BES}^0(3)$ process and a BES(3) bridge The paper is very interesting, it is remarkable that such explicit results are available, and may lead to further research on related problems.