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Title: Asymptotic behavior of a critical fluid model for a processor sharing queue via relative entropy.

MR Number: MR3633537

Primary classification:

Secondary classification(s):

Review text:

In a processor sharing queueing system the server allocates equal fraction of the service capacity to all items present in the queue at each point of time. Assuming that the arrival and service processes are random (of renewal type), the authors and co-authors have shown in previous work that a macroscopic description of the random queueing system is given by a deterministic evolution equation in the space of measures. Specifically, letting μ_t, $t \geq 0$, be a measure-valued function (by "measure" we mean Borel measure on $[0, \infty)$), the evolution equation is given by

$$\langle g, \mu_t \rangle = \langle g, \mu_0 \rangle + \alpha \langle g, \nu \rangle t - \int_0^t \frac{\langle g', \mu_s \rangle}{\langle 1, \mu_s \rangle} \, ds,$$

where $\langle g, \mu \rangle := \int_{[0,\infty)} g \, d\mu$. The measure ν is a probability measure and is derived from the arrival process. It is assumed that $\alpha = 1/\int x\nu(dx)$ and this is tantamount to "criticality". The measure μ_0 is the initial state. The test functions g are supposed to be bounded with bounded derivatives g' such that $g(0) = g'(0) = 0$. Previous papers focused on establishing this evolution equation (often known as "fluid limit" in the literature) from the random system, as well as properties of it. For example, the set of invariant states is completely characterized as follows. For a probability measure ζ on $[0, \infty)$ that has no atom at 0 and such that $\overline{\zeta} = \int x\zeta(dx) < \infty$ let ζ_e denote the probability measure

$$\zeta_e(dx) = \frac{1}{\overline{\zeta}} \left(\int_x^\infty \zeta(dy) \right) dx = p_{\zeta_e}(x) \, dx.$$

(So this measure has density $p_{\zeta_e}(x)$ with respect to the Lebesgue measure.) Then any measure of the form $\beta\nu_e$, $\beta \geq 0$, is an invariant measure for the fluid model. Let I be the set of these invariant measures. The paper under review establishes results about the asymptotic behavior, as $t \to \infty$, of the fluid model. Let d be the Prokhorov metric on the set of Borel measures on $[0, \infty)$. This is a standard metric that metrizes weak convergence on the space. If we let μ_t^ξ be the state of the fluid model at time t when $\mu_0 = \xi$, then the main theorem states that

$$d(\mu_t^\xi, I) \to 0, \text{ as } t \to \infty,$$

and the convergence is uniform with respect to certain compact sets. The proof of this theorem is the introduction of an entropy-type functional given by

$$H(\zeta) := \int_0^\infty h\left(\frac{p_{\zeta_e}(x)}{p_{\nu_e(x)}}\right) p_{\nu_e(x)}\, dx$$

where $h(x) := x \log x$, $x \geq 0$, $h(0) = 0$. The proof of the asymptotic behavior of the fluid model is based on the fact, also proved in the paper, that

$$H(\mu_t^\xi) \downarrow 0, \text{ as } t \to \infty,$$

again uniformly over certain compact sets.