This is a review text file submitted electronically to MR.

Reviewer: Konstantopoulos, Takis

Reviewer number: 68397

Address:

School of Mathematical & Computer Sciences
Heriot-Watt University
Edinburgh, EH14 4AS
UK
takis@ma.hw.ac.uk

Author: Purkayastha, Punyaslok; Baras, John S.

Short title: An optimal distributed routing algorithm using dual decomposition techniques.

MR Number: 2495877

Primary classification: 90C35

Secondary classification(s): 94C99

Review text:

This paper considers a macroscopic picture of a network modelled as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ with a specific subset of $\mathcal{N}$ designated as the set of destinations. Flow $r_i^{(k)} \geq 0$ enters node i exogenously and is destined for node k. (It is implicitly assumed that $r_k^{(k)} = 0$ for all destinations k.) To each directed link $(i,j) \in \mathcal{L}$ we associate a capacity C_{ij}. A flow $\mathbf{f}$ is an assignment of nonnegative numbers $f_{ij}^{(k)}$ to each link (i,j) and to each destination k. The usual flow balance relations

$$\sum_{j:(i,j)\in\mathcal{L}} f_{ij}^{(k)} = r_i^{(k)} + \sum_{j:(j,i)\in\mathcal{L}} f_{ji}^{(k)}$$

hold for each node i which is not a destination ($i \neq k$). The total flow on link (i,j) is $F_{ij} = \sum_k f_{ij}^{(k)}$. A measure of the congestion on link (i,j) is given by a function $G_{ij}(x)$ of the total flow x on this link; this is is assumed to be of integral type, i.e. $G_{ij}(x) = \int_0^x u D_{ij}(u)^\beta du$, where $D_{ij}(u)$ is an increasing $\mathcal{C}^1$ function on $0 \leq u < C_{ij}$ with $\lim_{u\to C_{ij}} D_{ij}(u) = \infty$. A prototype of such a function is $D_{ij}(u) = 1/(C_{ij} - u)$ which physically arises as the average delay in a simple (M/M/1) queue with arrival rate u and service rate C_{ij}. Typically, $\beta = 1$, but any $\beta > 0$ is considered. The problem solved (in the sense that an algorithm is provided) is to find the flow that minimizes $G(\mathbf{f}) := \sum_{(i,j)\in\mathcal{L}} G_{ij}(F_{ij})$ subject to the flow balance relations and to natural capacity constraints $F_{ij} < C_{ij}$ for each link (i,j). By defining the concave Legendre-Fenchel transform $Q_{ij}(p) := \inf_{0\leq x<C_{ij}}\{G_{ij}(x) - px\}$, $p \geq 0$, of each G_{ij} the dual problem becomes an unconstrained maximization problem. A

simple subgradient primal-dual algorithm converges to the unique optimizer. Numerical examples are given and the effect of the parameter β is investigated: as β increases, more flow is diverted towards the outgoing links that lie on paths with higher link capacities. A remark which provides further intuition is that the complementary slackness conditions can be interpreted as electrical circuit equations. Specifically, for the case of single-destination, the optimal solution (F_{ij}) of the primal problem and the optimal solution (p_i) of the dual problem satisfy the "nonlinear Ohm's law" $p_i - p_j = F_{ij} \cdot D_{ij}(F_{ij})^\beta$, provided that that the "potential difference" $p_i - p_j$ on link (i, j) is strictly positive; otherwise, if $p_i \leq p_j$, we have $F_{ij} = 0$. In other words, each link (i, j) is interpreted as a resistive diode where the "current" F_{ij} can flow only when the potential difference is positive and then the diode behaves like a resistor with nonlinear resistance $D_{ij}(F_{ij})^\beta$. The consideration of congestion function of integral type is motivated by the notion of Wardrop equilibrium which is a type of game-theoretic equilibrium specifically designed for traffic assignment problems.