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Review text:

The stationarity and stability of a spatial queueing system is studied in this paper. Particles arrive at times $s_1 < s_2 < \cdots$. The particle arriving at time s_n is placed in position X_n of a compact metric space (H, d). It also carries a tag $I_n \in \{-1, +1\}$. If $I_n = +1$ it is called a positive particle, otherwise negative. The state (or configuration) of the system, at each point of time, is described by the set of particle locations or, equivalently, by a point process (nonnegative, $\mathbb{Z} \cup \{+\infty\}$–valued Borel measure). The state evolves as follows: Initially, the state is N_0. When a positive particle arrives it is, simply, added to the current state. When a negative particle arrives in position $x \in H$, then it kills a point from the current configuration, chosen in some deterministic or randomized fashion from within those points y with $d(y, x) < 1$. Immediately, the arriving negative particle and the killed one vanish, so the new state has one point less. For example, the closest point is removed (locally greedy policy) or a point at random is killed (locally random policy). Other policies are also possible. Nothing happens between arrivals, i.e. the state remains constant. It is assumed that $\{(s_n; I_n, X_n)\}$ is a stationary (with respect to time-shifts) and ergodic marked point process under some probability measure $\mathbb{P}$. Moreover, the two random sequences $\{X_n^\pm\} := \{X_n : I_n = \pm 1\}$ are i.i.d., and independent of $\{I_n\}$. Let $\lambda_\pm$ be the rate of arrivals of positive (negative) particles. It is shown that, if $\lambda_+ < \lambda_-$, there exists a unique way to choose N_0 (as an element of the space of point processes) so that the the state process inherits the stationarity of the driving marked point process. Furthermore, under the additional assumption that H is a topological group (thus admitting a unique Haar probability measure), it is shown that, starting from an arbitrary initial configuration, the state of the system at time converges weakly to the stationary state. The author is careful in considering the possibility that the configuration may have accumulation points, showing that, using a nice argument in the proof of Theorem 1, that the accumulation points of the stationary configuration have a.s. empty interior. Examples with $\lambda_+ < \lambda_-$ and an infinite number of particles in the limit are also presented. One of the main tools of the paper is the Loynes' scheme, applicable due to a natural monotonicity property (noted in Lemma 1). In the last section, the author considers a multi-dimensional generalization of a spatial birth-death process (Robert (1987)).

Bibliography used in this review:

Philippe Robert (1987). Sur un processes de vie et de mort de particules sur $[0, 1]$. *Ann. IHP* **23**, 225-235.