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Title: Gibbs point process approximation: total variation bounds using Stein's method.

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A Gibbs point process on a compact metric space $\mathcal{X}$ equipped with a finite measure α is one whose law is defined by means of its Radon-Nikodym derivative u with respect to the law of a Poisson point process with mean measure α. Typically, the density u is given in an unnormalized form, i.e., up to a multiplicative constant which, in practice, may be hard to compute. The question then is whether meaningful bounds for the total variation distance between two Gibbs process can be found in terms of their unnormalized densities. Instead of working with an unnormalized density, one can work with ratios, in the form of the conditional intensity

$$\lambda(x|\xi) = \frac{u(\xi + \delta_x)}{u(\xi)},$$

where ξ is a finite point process (thought of as a measure with integer values) and δ_x is the Dirac measure with unit mass at $x \in \mathcal{X}$. The main theorem states that if Ξ and H are two Gibbs processes with conditional intensities $\nu(x|\xi)$ and $\lambda(x|\xi)$, respectively, then, provided that $\sup_\xi \int \lambda(x|\xi)\alpha(dx) < \infty$, the total variation distance $d_{\mathrm{TV}}(\Xi, \mathrm{H})$ satisfies

$$d_{\mathrm{TV}}(\Xi, \mathrm{H}) \leq c_1(\lambda) \int_{\mathcal{X}} \mathbb{E}|\nu(x|\Xi) - \lambda(x|\Xi)|\, \alpha(dx),$$

where $c_1(\lambda)$ is a constant depending on λ only. For the special case of pairwise interaction processes a more explicit bound is derived. Applications are provided to various well-known processes and settings from spatial statistics and

statistical physics, including the comparison of two Lennard-Jones processes, hard core approximation of an area interaction process and the approximation of lattice processes by a continuous Gibbs process. The proofs of the bounds are based on Stein's method. To obtain Stein factors, an explicit coupling between two spatial birth-death processes is constructed, and the Georgii-Nguyen-Zessin equation for the total bound is employed.