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Title: An SDE approach to leafwise diffusions on foliated spaces and its applications.

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This paper presents some new results in a series of papers around the idea that the leaves in a foliated Riemannian manifold can be thought of as the orbits of a dynamical system. L. Garnett (1983) considered a so-called leafwise Brownian motion along the leaves of a compact foliated manifold and showed existence of invariant measures. A. Candel (2003) showed that the corresponding semigroup possesses the Feller property. In this paper, K. Suzaki considers more general processes on a compact foliated manifold M called leafwise diffusions. Analytically, we are given a second-order differential operator A on M satisfying certain properties (mainly, positive definiteness of the coefficients of the second-order derivative) and ask whether there exists a Feller semigroup $T(t)$, $t \geq 0$, acting on functions on M and possessing an extension of A as generator. By reducing the problem to the study of a certain stochastic differential equation, the author gives a positive answer to this question. The main tool then is the study of Stratonovich-type stochastic differential equations of the form $dX(t) = \sum_{\alpha=1}^{r} A_\alpha(X(t)) \circ dB^\alpha(t) + A_0(X(t))dt$, where A_0 and the A_α are "leafwise smooth" vector fields on M (the notion being defined precisely in the paper) and the B^α are independent Brownian motions. It is shown that, starting from $x \in M$, a strong solution $X^x(t)$, $t \geq 0$, exists uniquely and that $X^x(\cdot)$ is stochastically continuous in x (and this implies the Feller property). As a corollary, it is easily seen that if m satisfies $\int_M (Af)dm$, for a certain class of functions $f : M \to \mathbb{R}$, then m is an invariant measure for $T(t)$. As another application, the author shows that a functional central limit theorem holds for additive functionals of the process $X^x(t)$, $t \geq 0$, of the form $Y^x_\lambda(t) = \lambda^{-1/2} \int_0^{\lambda t} f(X^x(s))ds$, namely that, as $\lambda \to \infty$, the process Y^x_λ converges in law to a Brownian motion and identifies the variance.