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Title: A new condition for the invariance principle for stationary random fields.

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Let $\epsilon = (\epsilon_k, k \in \mathbb{Z}^d)$, be a family of i.i.d. real-valued random variables indexed by the points of the d-dimensional integer lattice and let $f : \mathbb{R}^{\mathbb{Z}^d} \to \mathbb{R}$ be a measurable function such that $Ef(\epsilon) = 0$ and $E|f(\epsilon)|^p < \infty$ for some $p \geq 2$. if T_k is the operator on $\mathbb{R}^{\mathbb{Z}^d}$ that shifts the origin 0 to k then $f \circ T_k(\epsilon)$, $k \in \mathbb{Z}^d$, is a stationary random field. Let V_n be a sequence of rectangles of the form $[1, m_1^{(n)}] \times [1, m_d^{(n)}]$ with $m_i^{(n)} \to \infty$ for all i and define the partial sums process $S_n(f) := \sum_{k \in V_n} f \circ T_k$ over V_n. The paper establishes conditions under which the law of $S_n(f)/|V_n|^{1/2}$ converges weakly, as $n \to \infty$, to a normal random variable (central limit theorem or CLT for $S_n(f)$). The conditions are stated in terms of the filtration $\mathcal{F}_k := \sigma(\epsilon_\ell, \ell \leq k)$, where $\leq$ is the standard componentwise partial order on $\mathbb{Z}^d$. Specifically, it is proved that if

$$\widetilde{\Delta}_{d,p}(f) := \sum_{k \geq 1} (k_1 \cdots k_d)^{-1/2} E(|E(f \circ T_k | \mathcal{F}_1)|^p) < \infty,$$

for some $p \geq 2$, where $1 := (1, \ldots, 1)$, then the CLT for $S_n(f)$ holds. The weak limit is a zero mean normal random variable with variance $\sigma^2 = \lim ES_n(f)^2/|V_n|$. Moreover, if p is strictly bigger than 2, then a functional central limit theorem holds and the limit is a d-dimensional Brownian (or Wiener) sheet. The idea of the proof is the use of an approximating m-dependent sequence. Namely, if f_m is the conditional expectation of f given $\sigma(\epsilon_j, -m \leq j \leq m)$ then $f_m \circ T^k$, $k \in \mathbb{Z}^d$, is an m-dependent random field, in the sense that $f_m \circ T_k$ and $f_m \circ T_\ell$ are independent if the ℓ_∞ distance between k and ℓ exceeds m. For such a random field, a CLT exists (Bolthausen 1982). Moreover, f_m converges to f in

L^2. By using the idea of convergence in the "plus norm" (Zhao and Woodroofe 2008) and by showing that f_m converges to f in this plus norm, the proof of the (functional) CLT is established. The paper also establishes a (functional) CLT for orthomartingales and for functionals of stationary causal linear random fields (random fields obtained by linear operations on i.i.d. random variables in a "causal" manner).