Uppsala Universitet
Matematiska Institutionen
Thomas Erlandsson

DIFFERENTIAL GEOMETRY

PROBLEM 11

11. Let M be a Riemann surface with constant Gauss curvature $K = K_0$.

a) Calculate the circumference and area of a circle with radius r.

b) Calculate the geodesic curvature $k_g(r)$ of a circle with radius r. Determine $\lim_{r\to\infty} k_g(r)$.

c) Let $c(t)$ be a geodesic and $\gamma(t)$ a curve whose distance to $c(t)$ is constant $= d$. Calculate the geodesic curvature k_g of the equidistant curve γ.

RESULTS

WARNING! THERE ARE VERY LIKELY MISPRINTS HERE!

a)

$$L(C(r)) = \begin{cases} \dfrac{2\pi}{\sqrt{K_0}} \sin(r\sqrt{K_0}), & K_0 > 0 \\ 2\pi r, & K_0 = 0 \\ \dfrac{2\pi}{\sqrt{-K_0}} \sinh(r\sqrt{-K_0}), & K_0 < 0. \end{cases}$$

$$A(c(r)) = \begin{cases} \dfrac{\pi}{K_0}(1 - \cos(r\sqrt{K_0})), & K_0 > 0 \\ \pi r^2, & K_0 = 0 \\ -\dfrac{\pi}{K_0}(\cosh(r\sqrt{-K_0}) - 1), & K_0 < 0. \end{cases}$$

b)

$$k_g = \begin{cases} \sqrt{K_0}\cot(r\sqrt{K_0}), & K_0 > 0 \\ \dfrac{1}{r}, & K_0 = 0 \\ -\sqrt{-K_0}\coth(r\sqrt{-K_0}), & K_0 < 0. \end{cases}$$

c)

$$k_g = \begin{cases} -\sqrt{K_0}\tan(d\sqrt{K_0}), & K_0 > 0 \\ 0, & K_0 = 0 \\ \sqrt{-K_0}\tanh(d\sqrt{-K_0}), & K_0 < 0. \end{cases}$$