

CONCENTRATION COMPACTNESS
Functional-Analytic Grounds and Applications
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Corrections and Additions

CHAPTER 1

1.7 Differentiable functions

p.20 line 6 $u, v \in L$ should be $u, v \in V$.

CHAPTER 2

p. 29 line 9 $p = 2$ should be $m = 2$.

2.1 Weak derivatives

p.31 line 5 $C_0^\infty(\Omega)$ should be $C_0^1(\Omega)$.

2.3 Coordinate transformations. Trace domains...

p.34 line 12 $C^1(U \rightarrow V)$ should be $C^1(U, V)$.

2.7 Space $\mathcal{D}^{1,2}(\mathbb{R}^N)$...

p. 45 line -6 Insert “=” before the last integral!

p.46 line 11 (proof of Corollary 2.3): (2.34) should be (2.33)

CHAPTER 3

p. 59 line -9 “Of course bounded sequences...” should be “Bounded sequences...”

3.3 Weak convergence decomposition

p. 62 line -1 “be” should be “is”.

3.6 D -weak convergence with shift operators in \mathbb{R}^N

p. 73 line 5 $f(x, s)$ should be $F(x, s)$.

3.7 Constrained minimization

p.75 line 12 “derivaties” should be “derivatives”.

CHAPTER 4

4.9 Positive non-extremal solutions

p.104 lines 1,2 4,7 (all occurrences): $w^{(\infty)}$ should be $tw^{(\infty)}$.

4.10 Bibliographic remarks

An elaborate and refined penalty-based existence results in the subcritical case have been proved in:

Sirakov B., Existence and multiplicity of solutions of semi-linear elliptic equations in \mathbb{R}^N , Calc. Var. Partial Differential Equations 11 (2000), 119–142.

CHAPTER 5

5.1 Semilinear elliptic equations with the critical exponent

p.112 line 8: The exponent j in both occurrences has to be $j(N - 2)/2$.

5.7 Bibliographic remarks

Expansion of Theorem 5.1 applied to bounded sequences initial data for wave equation (resp. nonlinear Schrödinger equation) provides, in face of Strichartz estimates, expansions for corresponding solution sequences, as originally established in

Bahouri H., Gerard P., High frequency approximation of solutions to critical nonlinear wave equations, Amer. J.Math 121, 131–175 (1999),
resp. in Keraani, S., On the defect of compactness of the Strichartz estimates of the Schrödinger equations, J.Diff.Equ. 175 353–392 (2001).

Existence result similar to Theorem 5.2 but with the nonlinearity F satisfying a penalty condition ($F(s) \geq F_\infty(s)$ with strict inequality for s in a neighborhood of zero) is due to P.-L.Lions, [88]

CHAPTER 6

6.6 Bibliographic remarks

It can be shown that the statement Theorem 6.5 is equivalent to the existence result of Chabrowski and Yang,

Chabrowski J., Yang J., Existence theorems for elliptic equations involving supercritical Sobolev exponent, *Advances in Differential Equations*, 2 (1997), 231–256.

Theorem 6.8 can be derived as a corollary of the existence result of Flucher and Müller

Flucher M., Müller S., Concentration of low energy extremals, *Ann.Inst.H.Poincaré - Analyse non-linéaire* 16, (1999), 269-298

CHAPTER 9

9.4 Subelliptic mollifiers and Sobolev spaces on Carnot group

p. 211 line -9: Remove the line $+ \int |\partial_z u|^2 dx dy dz$

p. 212 line 3: $D^{1,2}(G)$ should be $H^1(G)$, although the original statement is also true.

p. 217 line 6: the last integral should be raised to the power 2.

p. 217 line 7: the norm of ψ should be raised to the power 2.

9.5 Compactness of subelliptic Sobolev imbeddings

p. 217 line -2: $M_t u_k$ - no need to restriction p. 217 line -2: Ω_1 should be Ω

9.7 Subelliptic Sobolev inequality

p. 222, line 4: exponent $-2_Q^* j$ in the second term should be $-(2_Q^* - 1)j$.

9.9 Concentration compactness on Carnot groups due to dilations

p. 224 line -8: $\delta_t \eta = \eta \delta_t$ has to be $\eta \delta_t \cdot = \delta_t(\delta_{t^{-1}} \eta \cdot)$