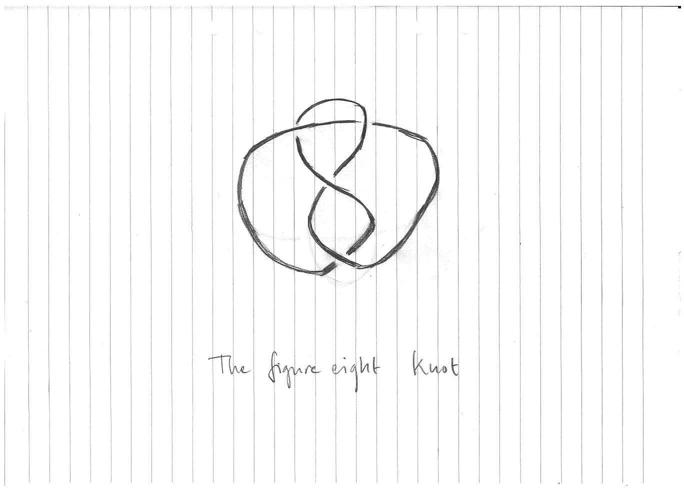
GEOMETRY/TOPOLOGY – PROBLEMS 2

1. Show that the figure eight knot shown below is isotopic to its mirror.



2. A knot diagram is n-*colorable* if each arc in the diagram (from one under crossing to another) can be colored by an element in $\mathbb{Z}_n$ such that at each crossing twice the color of the over arc equals the sum of the colors at the two under acrs and furthermore so that at least two different colors are used. Show that n-colorability is an isotopy invariant.

3. Show that the trefoil knot is 3-colorable (thereby proving it is knotted).

4. Describe the property of being n-colorable in terms of the knot group.

In problems **5** – **7** *below,* $V(L)$ *denotes the Jones polynomial of a link* L *defined as*

$$V(L) = \Big((-A)^{-3w(D)}\langle D\rangle\Big)_{t^{1/2}=A^{-2}} \in \mathbb{Z}[t^{-1/2}, t^{1/2}],$$

where $\langle D\rangle$ *is the Kauffman bracket of a diagram* D *for* L *and where* $w(D)$ *is the writhe of* D.

5. What is the value of $V(L)$ at $t = 1$?

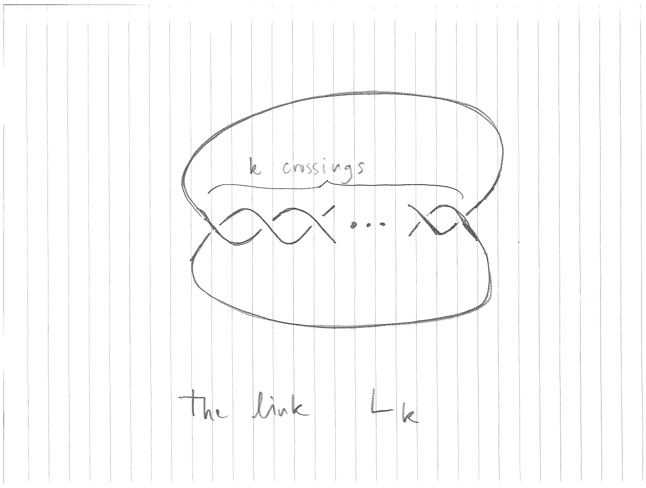
6. What is the value of $V(L)$ at $t^{1/2} = e^{2\pi i/3}$?

7. What is the value of $V(L)$ at $t^{1/2} = e^{\pi i/3}$?

8. Compute $V(L_k)$ where L_k is the link shown below.

9. Compute the Khovanov homology of the figure eight knot shown above.

10. Compute the Khovanov homology of the link L_k shown below.



11. If K_1 and K_2 are knots describe the Khovanov homology $\mathrm{Kh}(K_1 \# K_2)$ of the connected sum in terms of the Khovanov homologies $\mathrm{Kh}(K_1)$ and $\mathrm{Kh}(K_2)$ of the factors.