

5

5.1.

	0	1	2	
-1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$
1	0	$\frac{1}{2}$	0	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	1

5.2

berende, cell 1,1

$$\frac{1}{6} \neq \frac{1}{6} \cdot \frac{1}{2}$$

5.3

$$E(X) = \frac{2}{3} + 2 \cdot \frac{1}{6} = 1$$

$$E(Y) = 0$$

$$E(X^2) = \frac{2}{3} + 4 \cdot \frac{1}{6} = \frac{4}{3}$$

$$E(Y^2) = \frac{1}{2} + \frac{1}{2} = 1$$

$$V(X) = \frac{4}{3} - 1 = \frac{1}{3}$$

$$V(Y) = 1 - 0^2 = 1$$

$$E(X \cdot Y) = -1 \cdot \frac{1}{6} - 2 \cdot \frac{1}{6} + \frac{1}{2} = 0$$

$$C(X, Y) = 0 \quad \text{obostlerady}$$

5.4

$$V(X+Y) = V(X) + V(Y) + 2C(X, Y) = \frac{1}{3} + 1 = \frac{4}{3}$$

$$V(X-Y) = V(X) + V(Y) - 2C(X, Y) = \frac{4}{3}$$

6.

$R \in U(7,8)$

Uppgift 6

6.1.  $A = \pi \cdot R^2$  area  $f_R(r) = 1$

$$\begin{aligned} E(A) &= E(\pi \cdot R^2) = \int_7^8 \pi \cdot r^2 \cdot 1 \cdot dr = \frac{\pi}{3} \cdot r^3 \Big|_7^8 \\ &= \frac{\pi}{3} (8^3 - 7^3) = \frac{\pi}{3} \cdot 169 \end{aligned}$$

6.2  $E(A^2) = E(\pi^2 \cdot R^4) = \int_7^8 \pi^2 \cdot r^4 \cdot 1 \cdot dr = \frac{\pi^2}{5} r^5 \Big|_7^8$

$$= \frac{\pi^2}{5} (8^5 - 7^5) = \frac{\pi^2}{5} \cdot 15961$$

$$V(A) = E(A^2) - E(A)^2 = \frac{\pi^2}{5} \cdot 15961 - \frac{\pi^2}{9} \cdot 169^2$$

$$= \pi^2 \cdot 18.755 = 185 \cdot \pi = \underline{\underline{185}}$$

7

7.1  $X \in N(40, 10)$

$$\begin{aligned} P(X \leq 30) &= F_X(30) = \Phi\left(\frac{30-40}{10}\right) = \Phi(-1) = 1 - \Phi(1) \\ &= 1 - 0.8413 = \underline{\underline{16\%}} \end{aligned}$$

7.2  $X \in N(10, 2)$

$$\begin{aligned} P(X > 11.5) &= 1 - P(X \leq 11.5) = 1 - F_X(11.5) = \\ &= 1 - \Phi\left(\frac{11.5-10}{2}\right) = 1 - \Phi(0.75) = 1 - 0.7734 \approx \underline{\underline{23\%}} \end{aligned}$$

7.3  $X \in N(15, 5)$

$$\begin{aligned} P(12 < X \leq 18) &= F_X(18) - F_X(12) = \Phi\left(\frac{18-15}{5}\right) - \Phi\left(\frac{12-15}{5}\right) \\ &= \Phi\left(\frac{3}{5}\right) - \Phi\left(-\frac{3}{5}\right) = 2\Phi(0.6) - 1 \\ &= 2 \cdot 0.7257 - 1 \approx \underline{\underline{45\%}} \end{aligned}$$

7.4  $\bar{X}_{25} \in N(15, 1)$

$$\begin{aligned} P(12 < \bar{X}_{25} \leq 18) &= F_{\bar{X}}(18) - F_{\bar{X}}(12) = \Phi\left(\frac{18-15}{1}\right) - \Phi\left(\frac{12-15}{1}\right) \\ &= \Phi(3) - \Phi(-3) = 2\Phi(3) - 1 = 2 \cdot 0.998 - 1 \approx \underline{\underline{1\%}} \end{aligned}$$

7.5  $X \in N(100, 20)$     $Y \in N(110, 15)$     $n = 25$

$\bar{X} \in N(100, 4)$     $\bar{Y} \in N(110, 3)$

$\bar{Y} - \bar{X} \in N\left(10, \sqrt{4^2 + 3^2}\right)$   
 $\in N(10, 5)$

$$P(\bar{Y} - \bar{X} > 20) = 1 - P(\bar{Y} - \bar{X} \leq 20) = 1 - F_{\bar{Y}-\bar{X}}(20) = 1 - \Phi(2) =$$

8

A	A <sup>x</sup>
0.1	0.9
$\frac{R}{n}$	$1 - \frac{R}{n}$

$$n = 400$$

8.1.  $X \in \text{Bin}(400, 0.1)$

8.2.  $n \cdot p(1-p) = 400 \cdot 0.1 \cdot 0.9 = 36 > 10$  ja

8.3.  $P(35 < X \leq \frac{50}{\cancel{45}})$

$$X \in N(40, 6)$$

$$P(35 < X \leq 50) = F_X(50) - F_X(35)$$

$$= \Phi\left(\frac{50-40}{6}\right) - \Phi\left(\frac{35-40}{6}\right)$$

$$= \Phi(1.67) - \Phi(-0.83)$$

$$= \Phi(1.67) - 1 + \Phi(0.83)$$

$$= 0.9525 + 0.7967 - 1 = 0.75 = \underline{\underline{75\%}}$$

8.4

$$P(35 < X \leq 50) = \Phi\left(\frac{50.5-40}{6}\right) - \Phi\left(\frac{35.5-40}{6}\right)$$

$$= \Phi(1.75) - \Phi(-0.75)$$

$$= \Phi(1.75) + \Phi(0.75) - 1$$

$$= 0.9599 + 0.7734 - 1 = 0.73 = \underline{\underline{73\%}}$$