Homework I- Liese Course II deadline: 3/11/09

1. Consider two models
$$\mathcal{M} = \left(\mathcal{X}, \mathfrak{A}, \{P_\theta\}_{\theta\in\Theta}\right);\ \mathcal{N} = \left(\mathcal{Y}, \mathfrak{B}, \left\{P_\theta^{T(\mathbf{X})}\right\}_{\theta\in\Theta}\right)$$
Suppose $T(\mathbf{X})$ is a sufficient statistic for $\theta \in \Theta$. Show that the models are equivalent.

2. Consider two models $\mathcal{M}, \mathcal{N}$ and two equivalent models of them $\mathcal{M}_{suff}$, $\mathcal{N}_{suff}$ with $\Delta(\mathcal{M}, \mathcal{M}_{suff}) = 0,\ \Delta(\mathcal{N}, \mathcal{N}_{suff}) = 0$. Show
$$\Delta(\mathcal{M}, \mathcal{N}) = \Delta(\mathcal{M}_{suff}, \mathcal{N}_{suff})$$

3. Consider two models
$$\mathcal{M}_1 = \left(\mathbb{R}, \mathfrak{B}, \left\{N(\theta, \sigma_1^2\right\}_{\theta\in\mathbb{R}}\right);\ \mathcal{M}_2 = \left(\mathbb{R}, \mathfrak{B}, \left\{N(\theta, \sigma_2^2\right\}_{\theta\in\mathbb{R}}\right)$$
Derive an upper bound for $\Delta(\mathcal{M}_1, \mathcal{M}_2)$.

4. Consider two models
$$\mathcal{M}_1 = \left(\{0,1\}^N, \mathfrak{B}, \left\{Ber(p)^{\otimes N}\right\}_{p\in(0,1)}\right);\ \mathcal{M}_2 = \left(\{0,1\}^N, \mathfrak{B}, \left\{Ber(\pi p)^{\otimes N}\right\}_{p\in(0,1)}\right)$$
for some $\pi \in (0,1)$. Derive an upper bound for $\Delta(\mathcal{M}_1, \mathcal{M}_2)$.

5. Compare the models
$$\begin{aligned}\mathcal{M}_1 &= \left(\{0,1,...,n\}, \mathcal{P}(\{0,1,...,n\}), \{Bin(n,p)\}_{p\in(0,1)}\right)\\ \mathcal{M}_2 &= \left(\{0,1,...,n\}^3, \mathcal{P}\left(\{0,1,...,n\}^3\right), \left\{multinomial(n, \frac{p}{2}, \frac{p}{2}, (1-p)\right\}_{p\in(0,1)}\right).\end{aligned}$$

6. Compare the models
$$\begin{aligned}\mathcal{M}_1 &= \left(\{0,1,...,n\}, \mathcal{P}(\{0,1,...,n\}), \{Bin(n,p)\}_{p\in(0,1)}\right)\\ \mathcal{M}_2 &= \left(\{0,1,...,n\}^3, \mathcal{P}\left(\{0,1,...,n\}^3\right), \left\{multinomial(n, \frac{p}{3}, \frac{p}{3}, (1-\frac{2p}{3}\right\}_{p\in(0,1)}\right).\end{aligned}$$