

Homework II- Liese Course II deadline: 24/11/09

1. Consider two models

$$\mathcal{M}_1 = \left(\{0, 1\}^N, \mathfrak{B}, \{Ber(p)^{\otimes N}\}_{p \in (0,1)} \right); \mathcal{M}_2 = \left(\{0, 1\}^N, \mathfrak{B}, \{Ber(\pi p)^{\otimes N}\}_{p \in (0,1)} \right)$$

for some $\pi \in (0, 1)$. Derive a lower (!) bound for $\Delta(\mathcal{M}_1, \mathcal{M}_2)$.

2. Compare the models

$$\begin{aligned} \mathcal{M}_1 &= \left(\{0, 1, \dots, n\}, \mathcal{P}(\{0, 1, \dots, n\}), \{Bin(n, p)\}_{p \in (0,1)} \right) \\ \mathcal{M}_2 &= \left(\{0, 1, \dots, n\}^3, \mathcal{P}(\{0, 1, \dots, n\}^3), \left\{ multinomial\left(n, \frac{p}{3}, \frac{p}{3}, \left(1 - \frac{2p}{3}\right)\right) \right\}_{p \in (0,1)} \right). \end{aligned}$$

Derive a lower (!) bound for $\Delta(\mathcal{M}_1, \mathcal{M}_2)$

3. Consider

$$\mathcal{M} = \left(\{0, 1\}^N, \mathfrak{B}, \{Ber(p)^{\otimes N}\}_{p \in (0,1)} \right)$$

as original model and a bootstrap model, for the resampled observations (nonparametric bootstrap) from \mathcal{M}_1 .

- (a) Formulate the bootstrap model for the resampled variables

$$\mathcal{M}^* = \left(\{0, 1\}^{NB}, \mathfrak{B}^*, \{P_p^*\}_{p \in (0,1)} \right)$$

- (b) Derive an upper bound for $\Delta(\mathcal{M}, \mathcal{M}^*)$. ($B = 1$)
- (c) Derive a lower bound for $\Delta(\mathcal{M}, \mathcal{M}^*)$.
- (d) Are the models \mathcal{M} and \mathcal{M}^* equivalent?
- (e) Are the models asymptotically equivalent?