Homework III- Liese Course II deadline: 8/12/09

1. Consider the model

$$\mathcal{M} = (\{0,1,2,3\}, \mathcal{P}(\{0,1,2,3\}), \{P_1, P_2, P_3\})$$

with

x	0	1	2	3
P_1	$\frac{1}{2}$	$\frac{1}{2}$	0	0
P_2	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
P_3	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(a) Derive the standard model and the standard distribution.

(b) Formulate the models

$$\mathcal{M}_\alpha = (\{0,1,2,3\}, \mathfrak{B}, \{P_{1,\alpha}, P_{2,\alpha}, , P_{3,\alpha}\}) \ , \ \alpha \in [0,1]$$

(c) Calculate

$$H_s(P_{1,\alpha}, P_{2,\alpha}, , P_{3,\alpha})$$

2. Construct a series of models

$$\mathcal{M}_n = \left([0,3], \mathfrak{B}_{[0,3]}, \{P_{1,n}, P_{2,n}, P_{3,n}\}\right)$$

such that $\lim_{n\to\infty} \Delta(\mathcal{M}, \mathcal{M}_n) = 0$, for $\mathcal{M}$ given in 1.

3. Consider the models

$$\begin{aligned} \mathcal{M}_n &= \left(\mathbb{R}, \mathfrak{B}, \left\{Exp\left(\lambda + \frac{1}{\sqrt{n}}h\right)\right\}_{\lambda\in(0,10], h\in[0,1]}\right) \\ \mathcal{M} &= \left(\mathbb{R}, \mathfrak{B}, \{Exp(\lambda)\}_{\lambda\in(0,10]}\right) . \end{aligned}$$

Show (!, directly for this case, with help of Theorem 6.13) that $\Delta(\mathcal{M}_n, \mathcal{M}) \to 0$.

4. (* facultative) Consider the

simple linear regression model for $(y_i, x_i), i = 1, .., n$

$$y_i = \beta x_i + \varepsilon_i,\ i = 1..n,\ \varepsilon_i\ i.i.d.\ N(0,1),\ x_i\ i.i.d.\ N(0,1)$$

and the simple linear errors - in- variable model for $(y_i, x_i), i = 1, .., n$

$$y_i = \beta\xi_i + \varepsilon_i,\ x_i = \xi_i + \delta_i,\ \ \varepsilon_i\ i.i.d\ N(0,1),\ \xi_i\ i.i.d.\ N(0,1),\ \delta_i\ i.i.d.\ N(0,1)$$

(a)Formulate both as

$$\mathcal{M}_n = \left(\mathbb{R}^{2n}, \mathfrak{B}, \left\{P_\beta^n\right\}_{\beta\in\mathbb{R}}\right)$$

and

$$\mathcal{Y}_n = \left(\mathbb{R}^{2n}, \mathfrak{B}, \left\{Q_\beta^n\right\}_{\beta\in\mathbb{R}}\right).$$

(b)Find the models related to the sufficient statistics.

(c) Discuss the distance

$$\Delta\left(\mathcal{Y}_n, \mathcal{M}_n\right).$$