ANALYTIC NUMBER THEORY: THE ORAL EXAM

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The following things I would like you to know and be able to tell me and write on the board:

• The content of every definition and every theorem in the text, excluding "external reading" sections marked by " * ".

Exceptions:

× In Section 4.7, I'll be satisfied with just Def 4.7, Def 4.8, Thm 4.23, Thm 4.24 plus you should "be aware" of the remaining definitions and theorems even if you perhaps cannot state them in exact details.

× In Section 5, I only ask that you learn Theorem 5.4 and all definitions needed to define $h(d)$, and Lemma 5.15; you can skip *all* other definitions and theorems if you want.

× You don't have to learn the exact statements of the theorems in Section 6 (since you anyway have to learn more precise theorems from later sections).

× In Section 7, I won't ask on Theorems 7.5, 7.6 (although I think you ought to know these anyway).

× In Section 9 you can skip Theorem 9.2.

× Regarding Theorem 13.2, perhaps you don't have to remember the exact form of the bound on $R(x, T)$, so long as you remember "the main points" of the theorem.[1]

× Regarding Theorem 14.1 you don't have to remember exactly the terms "$-(1-a)\log x - b(\chi) + \sum_{m=1}^{\infty} \frac{x^{a-2m}}{2m-a}$", nor the exact form of the bound on $R(x, T)$ (cf. the footnote).

× In Theorem 19.2 perhaps you don't need to remember the exact formula for $\mathfrak{S}(N)$ (so long as you remember that $\mathfrak{S}(N) = 0$ for N even and that, for N odd, $\mathfrak{S}(N)$ is bounded from above and below by some absolute positive constants).

• Proposition 2.7 (and especially what it gives for $\zeta(s)$ and $L(s, \chi)$).

• Lemma 3.2 (and to give some example of its usage in practice).

• Corollary 3.7.

• Proposition 3.10 (and to give some example of its usage in practice).

• Lemma 3.13 and its consequence regarding σ_c for $L(s, \chi)$.

• (113), (114), (115), and in particular the special case of these for $\zeta(s)$.

• Lemma 4.13 and Lemma 1.5 and how the latter is used get down to a single congruence class modulo q, e.g. in (23) and (508).

• Lemma 4.21 together with the definition of "induce" on p.63 and Lemma 11.7.

• Lemma 4.22.

• Proposition 6.2.

• Proposition 8.3.

• Lemmas 8.10, 8.11, 8.12, 8.13, 8.14. Proposition 8.18.

• Corollary 9.3, Lemma 9.4, Lemma 9.5, Corollary 9.7.

• Proposition 13.10.

• Proposition 20.5.

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[1] Thus: I'd be happy if you'd write out "$\psi_0(x) = x - \sum_{|\gamma|<T} \frac{x^\rho}{\rho} - \frac{\zeta'(0)}{\zeta(0)} - \frac{1}{2}\log(1 - x^{-2}) + O(***)$" and then say "where *** is an error term which is uniform in both x and T, and which is good for applications". Or even better if you could *also* say: "so long as x is an integer ((so that $\langle x \rangle \geq 1$)), the error term $***$ equals $\frac{x}{T}$ times some log factor".

Furthermore, I may ask you to *say a few words about the proof* of any of the above results, but note that I may consider your performance to be very good even if you *fail* to answer several of these questions!

By "say a few words" I mean: Tell me some sentences to describe what happens in the proof, and *ideally* also write out the 1-3 most important steps or formulas in the proof. (For example, if I'd ask you to say a few words about the proof of the prime number theorem, I would be happy if you write out formula (290) and then just say a few sentences about how this is used to reach the goal, especially pointing out the importance of the fact that $\zeta(s)$ does not have any zeros along $\operatorname{Re} s = 1$.)

Here follows a list of results for which I am extra likely to ask you to "say a few words about the proof":

- Dirichlet's Theorems 1.1, 1.4.
- The prime number theorem, Theorem 7.1.
- The functional equation of $\zeta(s)$, Theorem 9.1.
- The zero-free region for $\zeta(s)$, Theorem 11.1.
- von Mangoldt's formula, Theorem 12.1-2.
- Siegel's Theorem 16.1-2.
- The Polya-Vinogradov Inequality, Theorem 17.1.
- Vinogradov's 3-primes theorem; Theorem 19.1-2. (Here I will in principle be satisfied if you just write out (601) and (605) and then say 2-3 sentences about how the integral (605) is treated, without necessarily writing out anything explicit on the board.)