Uppsala University
Department of Mathematics
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Examination in mathematics
2022-06-09

# Analytic Number Theory

*Time: 14.00 – 19.00. No tools are allowed except paper and pen.*

**1.** Define the Jacobi Theta Function $\Theta(z \mid \tau)$ which we studied in the course. (5p)

**2.** State the defining formula of the Dirichlet $L$-function $L(s, \chi)$ as a Dirichlet series, and also the Euler product formula for $L(s, \chi)$. Outline a proof of the fact that the two expressions are equal. (You may work formally and do not need to discuss convergence.) (5p)

**3.** Define the $\Gamma$-function, and explain how this function provides a meromorphic continuation to all $\mathbb{C}$ of the arithmetic function $n \mapsto n!$.
(5p)

**4.** Determine (with proof) whether the two integral binary quadratic forms $x^2 + y^2$ and $2x^2 - 2xy + y^2$ are equivalent or not. (5p)

**5.** Outline a proof of the prime number theorem.

(Note: You do not need to prove anything, instead merely describe the main steps of a proof.) (10p)

**6.** State Rényi's large sieve bound (that is, a certain bound on the cardinality of a set of integers, when we assume that this set is contained in a certain interval, and satisfies certain congruence conditions), and describe the main steps in a proof of this result. (10p)

**LYCKA TILL / GOOD LUCK!**