Information and Coding Theory: Tenta 2002-08-17

Allowed material: Calculators, the text book and notes.

1.) Consider the sequences $(2,2,3,3,3,3,4)$ and $(2,2,2,4,4,4,4)$ and decide whether there is a prefix-free binary code having the numbers in those sequences as word lengths. If yes, determine such a code!

2.) Let $\mathcal{S}$ be a source with 6 source symbols with probabilities $0.4, 0.2, 0.2, 0.1, 0.06, 0.04$. Determine the binary entropy $H(\mathcal{S})$ and the average word lengths of a (binary) optimal code and a (binary) Shannon-Fano code of $\mathcal{S}$!

3.) Let $\mathcal{S}$ be a source having source symbols $s_1, ..., s_8$ with probabilities $0.35, 0.2, 0.15, 0.1, 0.1, 0.05, 0.03, 0.02$. Compute the ternary entropy $H(\mathcal{S})$ and the average word length of an optimal ternary code for $\mathcal{S}$. List all code words in such a code!

4.) List all code words in a ternary Hamming code $\mathcal{H}_4 \subset F_3^4$!

5.) Let $\mathcal{A}$ and $\mathcal{B}$ be sources with alphabet a_1, a_2 resp. b_1, b_2 respectively, Γ a channel between $\mathcal{A}$ and $\mathcal{B}$ with channel matrix

$$M := \begin{pmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix} .$$

Let a_1, a_2 have the probabilities 0.2 resp. 0.8. Compute the probabilities for the output symbols b_1, b_2 as well as the backward probabilities Q_{ij}, the binary system entropies $H(\mathcal{A}, \mathcal{B}), H(\mathcal{A}|\mathcal{B}), H(\mathcal{B}|\mathcal{A})$ and the mutual information $I(\mathcal{A}, \mathcal{B})$!

6.) Let $\mathcal{A}$, $\mathcal{B}$ be sources with source alphabets $A = B := \mathbf{Z}_3 = \{0, 1, 2\}$ and Γ be the channel from $\mathcal{A}$ to $\mathcal{B}$ with matrix

$$M := \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{pmatrix} .$$

Assume that $0, 1, 2$ are emitted by $\mathcal{A}$ with the probabilities $\frac{1}{8}, \frac{5}{8}, \frac{1}{4}$. For the repetition code $\mathcal{R}_2 \subset A^2$ compute an ideal observer and a maximal likelihood rule $\Delta : B^2 \longrightarrow \mathcal{R}_2$ together with the corresponding error probabilities $\Pr_E$!

7.) Let $F = F_q$ and $n = \frac{q^c - 1}{q - 1}$. Compute the minimum distance $d(\mathcal{H}_n)$ of a Hamming code $\mathcal{H}_n \subset F^n$ and show that it is perfect.

8.) Show that for a perfect code the minimum distance $d(\mathcal{C})$ is odd.

LYCKA TILL!