Information and Coding Theory: Tenta 2003-01-08

Allowed material: Calculators, the text book and notes.

1.) For a given source alphabet S and code alphabet T define uniquely decodable resp. prefix-free (=instantaneous) codes and compare them with respect to word lengths and decoding!

2.) Is there a uniquely decodable ternary code having word lengths $(1, 2, 3, 4, 5, 6)$? If yes, determine such a code!

3.) Let $\mathcal{S}$ be a source with 5 source symbols with probabilities $0.4, 0.2, 0.2, 0.1, 0.1$. Determine the binary entropy $H(\mathcal{S})$ and the average word lengths of a (binary) optimal code and a (binary) Shannon-Fano code of $\mathcal{S}$!

4.) Let $\mathcal{S}$ be a source having source symbols $s_1, ..., s_6$ with probabilities $0.3, 0.2, 0.15$, $0.15, 0.1, 0.1$. Compute the ternary entropy $H(\mathcal{S})$ and the average word length of an optimal ternary code for $\mathcal{S}$. List all code words in such a code!

4.) Explain the notion of a linear code! How can the minimum distance of such a code be computed? Prove your statement!

5.) Let $F := F_5$. Determine a parity check matrix $H \in F^{2,6}$ and a generator matrix $G \in F^{4,6}$ for a Hamming code $\mathcal{H}_6 \subset F^6$.

6.) Given a source S with probabilities for the source symbols and a code alphabet T with r symbols, explain the relationship between the r-ary entropy of S and average word lengths of certain encodings!

7.) Let $F := F_2$ and $\mathcal{C} \subset F^{23}$ be a perfect code with 2048 equiprobable codewords. Compute the minimum distance $d(\mathcal{C})$!

8.) Show or give a counter example for the following statement: "Let $\mathcal{C}$ be an optimal binary code for a source with alphabet $s_1, ..., s_q$ and probabilities $p_1 \geq p_2 \geq ... \geq p_q > 0$, such that the word lengths $\ell_i := \lambda(\mathcal{C}(s_i))$ satisfy $\ell_1 \leq \ell_2 \leq ... \leq \ell_q$. Then the vector $(\ell_1, ..., \ell_q)$ depends only on the probabilities $p_1, ..., p_q$."

LYCKA TILL!