Information and Coding Theory: Tenta 2002-05-31

Allowed material: Calculators, the text book and notes.

1.) Consider the sequences $(2, 2, 3, 3, 3, 4, 4)$ and $(2, 2, 2, 3, 3, 4, 4)$ and decide whether there is a prefix-free binary code having the numbers in those sequences as word lengths. If yes, determine such a code!

2.) Let $\mathcal{S}$ be a source with 6 source symbols with probabilities $0.3, 0.3, 0.2, 0.1, 0.08, 0.02$. Determine the binary entropy $H(\mathcal{S})$ and the average word lengths of a (binary) optimal code and a (binary) Shannon-Fano code of $\mathcal{S}$!

3.) Let $\mathcal{S}$ be a source having source symbols $s_1, ..., s_8$ with probabilities $0.3, 0.25, 0.15, 0.1, 0.1, 0.04, 0.03, 0.03$. Compute the ternary entropy $H(\mathcal{S})$ and the average word length of an optimal ternary code for $\mathcal{S}$. List all code words in such a code!

4.) Let $T := \mathbf{Z}_r := \{0, 1, ..., r-1\}$. Let $\mathcal{C} \subset T^*$ be a prefix-free code with word lengths $\ell_1, ..., \ell_q$. Call $\mathcal{C}$ exhaustive (or transversal) iff any infinite sequence of symbols $\in T$ admits an initial segment $\in \mathcal{C}$. Show that $\mathcal{C}$ is exhaustive iff $\sum_{i=1}^{q} r^{-\ell_i} = 1$.

5.) Let $\mathcal{A}$ and $\mathcal{B}$ be sources with alphabet a_1, a_2 resp. b_1, b_2 respectively, Γ a channel between $\mathcal{A}$ and $\mathcal{B}$ with channel matrix

$$M := \begin{pmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{pmatrix} .$$

Let a_1, a_2 have the probabilities 0.2 resp. 0.8. Compute the probabilities for the output symbols b_1, b_2 as well as the backward probabilities Q_{ij}, the binary system entropies $H(\mathcal{A}, \mathcal{B}), H(\mathcal{A}|\mathcal{B}), H(\mathcal{B}|\mathcal{A})$ and the mutual information $I(\mathcal{A}, \mathcal{B})$!

6.) Let $F := F_3$ and $\mathcal{C} \subset F^{11}$ be a code with 729 equiprobable codewords and minimum distance $d(\mathcal{C}) = 5$.
i) Show that $\mathcal{C}$ is perfect.
ii) Let Γ be the binary symmetric channel with the probability $Q := 0.1$ that a digit 0 or 1 is not transmitted correctly. For the nearest neighbour decoding $\Delta : F^{11} \longrightarrow \mathcal{C}$ compute the error probability that a transmitted code word is not decoded correctly!

7.) Let $F := F_5$. Determine a parity check matrix $H \in F^{2,6}$ and a generator matrix $G \in F^{4,6}$ for a Hamming code $\mathcal{H}_6 \subset F^6$.

8.) Let $F = F_q$ and $\mathcal{C} \subset F^n$ a code with q^{n-3} code words and minimum distance $d(\mathcal{C}) = 3$. Show that $n \leq q^2 + q + 1$.

LYCKA TILL!