CORRECTIONS TO “FREE PATH LENGTHS IN QUASICRYSTALS” AND “VISIBILITY AND DIRECTIONS IN QUASICRYSTALS”

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Abstract. We provide some corrections to the published articles [1] and [2].

- The definition of the Siegel transform, [1, (5.1)], should read as follows:

$$\widehat{f}(\Gamma h) = \sum_{\substack{\boldsymbol{m}\in\mathbb{Z}^n hg \\ \pi(\boldsymbol{m})\neq\boldsymbol{0}}} f(\boldsymbol{m}).$$

The proof of [1, Theorem 5.1] has to be corrected by replacing each occurrence of “$\setminus\{\boldsymbol{0}\}$” (except the one in p. 747, line 4) by “$\setminus\pi^{-1}(\{\boldsymbol{0}\})$”. Also in the proof of [1, Cor. 5.2], each occurrence of “$\setminus\{\boldsymbol{0}\}$” should be replaced by “$\setminus\pi^{-1}(\{\boldsymbol{0}\})$”, and also, on p. 747, line -3, “$\boldsymbol{0}\notin\mathbb{Z}^n hg+\boldsymbol{z}$” should be replaced by “$\pi^{-1}(\boldsymbol{0})\cap(\mathbb{Z}^n hg+\boldsymbol{z})=\emptyset$”.

- In [2, (4.1)], “$\sum_{\boldsymbol{q}\in\mathcal{P}^x}$” should be replaced by “$\sum_{\boldsymbol{q}\in\mathcal{P}^x\setminus\{\boldsymbol{0}\}}$”.

- In [2, line -5 of p. 6600], “$\mu(S_1)=0$” need not hold; cf. the correction of [1, Theorem 5.1]. To correct the proof of [2, Lemma 9], we first prove:

Lemma 1. *If $\boldsymbol{m}\in\mathbb{Z}^n$ and $\pi(\boldsymbol{m}g)=\boldsymbol{0}$, then $\boldsymbol{m}h=\boldsymbol{m}$ for all $h\in H_g$.*

Proof. Let $h\in H_g$ be given. Because of $H_g\subset\Gamma H_g=\overline{\Gamma\varphi_g(\mathrm{SL}(d,\mathbb{R}))}$ (closure in $G=\mathrm{ASL}(n,\mathbb{R})$), there exist $\gamma_1,\gamma_2,\ldots\in\Gamma$ and $a_1,a_2,\ldots\in\mathrm{SL}(d,\mathbb{R})$ such that $\gamma_j\varphi_g(a_j)\to h$ as $j\to\infty$. It follows from $\pi(\boldsymbol{m}g)=\boldsymbol{0}$ that $\boldsymbol{m}g\left(\begin{smallmatrix} a_j & 0 \\ 0 & 1_m\end{smallmatrix}\right)^{-1}=\boldsymbol{m}g$ for all j, and thus $\boldsymbol{m}(\gamma_j\varphi_g(a_j))^{-1}=\boldsymbol{m}\gamma_j^{-1}\in\mathbb{Z}^n$. However $\boldsymbol{m}(\gamma_j\varphi_g(a_j))^{-1}\to\boldsymbol{m}h^{-1}$ as $j\to\infty$, and hence, since $\mathbb{Z}^n$ is discrete, $\boldsymbol{m}h^{-1}\in\mathbb{Z}^n$. This is true for all $h\in H_g$, and since H_g is connected it follows that $\boldsymbol{m}h^{-1}$ is independent of $h\in H_g$; thus $\boldsymbol{m}h^{-1}=\boldsymbol{m}e^{-1}=\boldsymbol{m}$ for all $h\in H_g$. □

Now replace the first paragraph of the proof of [2, Lemma 9] by the following: “Set $M_E=\{\boldsymbol{m}\in\mathbb{Z}^n\,:\,\pi(\boldsymbol{m}hg)=\boldsymbol{0}$ for all $h\in H_g\}$ and $(\mathbb{Z}^n)':=\mathbb{Z}^n\setminus M_E$. Note that by [2, Lemma 8], if $\boldsymbol{m}\in(\mathbb{Z}^n)'$ then $\pi(\boldsymbol{m}hg)\neq\boldsymbol{0}$ for almost all $h\in H_g$.” The remainder of the proof of [2, Lemma 9] is kept unchanged (in particular S_1 is defined as before, but with our new “$(\mathbb{Z}^n)'$”), except for two modifications: Replace the third sentence below (7.5) by: “Also for every $\boldsymbol{m}\in F\cap M_E$ we have $\boldsymbol{m}h'g=\boldsymbol{m}g=\boldsymbol{m}hg\in U\times\mathcal{W}$ for *all* $h'\in H_g$.” In the first sentence in the last paragraph of the proof, replace “cannot have $\boldsymbol{m}=\boldsymbol{m}_E$” by “cannot have $\boldsymbol{m}\in M_E$ (by Lemma 1)”.

(Let us add some details for the proof of the fact $\mu(S_2)=0$ stated below [2, (7.3)]: The set $S_2':=\big\{h\in H_g\,:\,\exists\boldsymbol{\ell}_1\neq\boldsymbol{\ell}_2\in\mathbb{Z}^n hg\cap\pi_{\mathrm{int}}^{-1}(\mathcal{W}^\circ)$ satisfying $\pi(\boldsymbol{\ell}_1)=\pi(\boldsymbol{\ell}_2)\big\}$ has measure zero, by [1, Proposition 3.7]. Also the set $S_2''=\big\{h\in H_g\,:\,(\mathbb{Z}^n)'hg\cap\pi^{-1}(\{\boldsymbol{0}\})\neq\emptyset\big\}$ has measure zero, by [2, Lemma 8]. Hence it suffices to prove $S_2\subset S_1\cup S_2'\cup S_2''$. Assume $h\in S_2\setminus(S_1\cup S_2'\cup S_2'')$, and take $\boldsymbol{\ell}_1\neq\boldsymbol{\ell}_2\in\mathbb{Z}^n hg\cap\pi_{\mathrm{int}}^{-1}(\mathcal{W})$ satisfying $\pi(\boldsymbol{\ell}_1)=\pi(\boldsymbol{\ell}_2)$. Because of $h\notin S_2'$, either $\boldsymbol{\ell}_1$ or $\boldsymbol{\ell}_2$ must lie in $\pi_{\mathrm{int}}^{-1}(\partial\mathcal{W})$; say $\boldsymbol{\ell}_1\in\pi_{\mathrm{int}}^{-1}(\partial\mathcal{W})$. Then, using $h\notin S_1$, there is $\boldsymbol{m}_1\in M_E$ so that $\boldsymbol{\ell}_1=\boldsymbol{m}_1hg=\boldsymbol{m}_1g$ (cf. Lemma 1); thus also $\pi(\boldsymbol{\ell}_2)=\pi(\boldsymbol{\ell}_1)=\boldsymbol{0}$. Using $h\notin S_2''$ it follows that also $\boldsymbol{\ell}_2=\boldsymbol{m}_2hg=\boldsymbol{m}_2g$ for some $\boldsymbol{m}_2\in M_E$. This gives a contradiction to the assumption

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that the projection map from $\mathbb{Z}^n g \cap \pi_{\text{int}}^{-1}(\mathcal{W})$ to $\mathcal{P} = \mathcal{P}(\mathcal{W}, \mathbb{Z}^n g)$ is injective, and the proof is complete.)

• Also [2, Lemma 10] is still true, and the proof holds as written, so long as in the first sentence we understand M_E, $(\mathbb{Z}^n)'$, S_1 and S_2 to be as in our corrected proof of [2, Lemma 9].

• [2, Lemma 11] is correct as stated but there are some corrections required in the proof. In the first line of the proof, the definition of $\widetilde{\mathcal{P}}'$ should be replaced by $\widetilde{\mathcal{P}}' = \mathcal{P}' \setminus (\widehat{\mathcal{P}}' \cup \{\mathbf{0}\})$. This is needed to make the equality on the right of [2, (8.5)] as well as the statement on [2, p. 6605 (line 1)] correct. The rest of the proof of the lemma is not affected by the modified definition of $\widetilde{\mathcal{P}}'$.

Also near the end of the proof of the lemma, the equality in [2, (8.11)] is incorrect (a term $\big[-\tau^d A_R(\tau)\big]_{\tau=(1-\eta)\tau_j}^{\tau=\tau_j}$ is missing), but the overall conclusion in [2, (8.11)], that $\int_{(1-\eta)\tau_j}^{\tau_j} \tau^d(-dA_R(\tau))$ tends to a positive limit as $j \to \infty$, still holds. One way to prove this to assume $\eta < \frac{1}{2}$ as we may do without loss of generality, then note that

$$\int_{(1-\eta)\tau_j}^{\tau_j} \tau^d(-dA_R(\tau)) \geq (\tfrac{1}{2}\tau_j)^d \big(A_R((1-\eta)\tau_j) - A_R(\tau_j)\big),$$

and finally use the fact that for each fixed $\beta \in (0,1)$, $A_R(\beta R^{d-1}) \sim f(\beta) R^{-d(d-1)}$ as $R \to \infty$, where $f(\beta)$ is a strictly decreasing function of β.

• In the last sentence of [2, Section 9] (the proof of Theorem 1), "Theorem 6" should be corrected to "Theorem 1".

• [2, p. 6610, line -12]; replace "$d \geq 1$" by "$d \geq 2$".

References

[1] J. Marklof and A. Strömbergsson, Free path lengths in quasicrystals, Communications in Mathematical Physics **330** (2014), 723-755.

[2] J. Marklof and A. Strömbergsson, Visibility and Directions in Quasicrystals, Int. Math. Res. Notices **2015** (2015), 6588-6617.

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