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ÖVNINGAR, Blad 1
Transformer för beräkningar
2000 04 07 00trp1

1.1. Suppose we know that $a \in l^2(\mathbf{Z}_N)$ satisfies $\hat{a}(0) = 0$, $\hat{a}(k) \neq 0$, $k = 1, ..., N-1$. Can you determine the dimension of

$$\{z \in l^2(\mathbf{Z}_N);\ a * z = 0\}?$$

Can you determine the dimension of

$$\{a * z;\ z \in l^2(\mathbf{Z}_N)\}?$$

Generalize to an arbitrary convolution equation $a * z = b$, where a, b are given vectors, z an unknown vector.

1.2. Calculate $\hat{z}$ when $z = (1, 2, 3, 4)^{\mathrm{T}} \in l^2(\mathbf{Z}_4)$. Calculate $\widehat{w}$ when

$$w = (1, 2, 3, 4, 5, 6)^{\mathrm{T}} \in l^2(\mathbf{Z}_6).$$

1.3. Define a mapping $T: l^2(\mathbf{Z}_4) \to l^2(\mathbf{Z}_4)$ by

$$T(z) = (2z(0) - z(1), iz(1) + 2z(2), z(1), 0).$$

Is T linear? Is T translation invariant? Calculate $T(R_1 z)$ and $R_1(Tz)$ for $z = (1, 0, -2, i)^{\mathrm{T}}$.

1.4. Let S, T be two translation-invariant linear mappings of $l^2(\mathbf{Z}_N)$ into itself. Prove that they commute, i.e., that $T \circ S = S \circ T$. Deduce that circulant matrices always commute—try also to prove this fact by a direct calculation using the definition of circulant matrices.

1.5. Prove that $T(z * w) = T(z) * w = z * T(w)$ if T is a translation-invariant linear mapping of $l^2(\mathbf{Z}_N)$ into itself.

1.6. Prove that a linear mapping T of $l^2(\mathbf{Z}_N)$ into itself is translation invariant if and only if it is a polynomial in the translation operator R_1.

1.7. Let $u = (1, 3)^{\mathrm{T}}$, $v = (0, 4)^{\mathrm{T}}$, $z = (1, 0, 3, 4)^{\mathrm{T}}$. Compute $\hat{u}$, $\widehat{w}$ and then $\hat{z}$ using the fast Fourier transform. Compute the Fourier transform of $w = (0, 1, 4, 3)^{\mathrm{T}}$.

1.8. Compute the Fourier transform of $w = (1, 0, 1, 0, 1, 0, 1, 0)^{\mathrm{T}}$—maybe using $z = (1, 1, 1, 1)^{\mathrm{T}}$.